Mechanics
MS Comprehensive Exam Spring 2020

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1. Consider a linear elastic material with stress-strain relation

$$\epsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij}$$

   (a) Derive an expression for the bulk modulus of the material in terms of the Young's modulus and the Poisson ratio.

   (b) Explain what happens in the limit $\nu \to 0$.

2. Consider a cuboidal body $\Omega = [0, L] \times [0, L] \times [0, L]$, with known $L > 0$. The body is observed to be deformed with displacement field

$$u_1 = \alpha x_1 \qquad u_2 = \gamma x_1 \qquad u_3 = 0\,,$$

   where $\alpha > 0$ and $\gamma > 0$ are known measured values.

   (a) Find the strain field.

   (b) Assume linear elastic isotropic behavior with Lamé constants (λ, μ), find the stress field.

   (c) Assuming a yield stress Y in tension, determine the relation between α and γ at yield according to the Mises condition.

UNIVERSITY OF CALIFORNIA AT BERKELEY
SPRING SEMESTER 2018

Department of Civil and Environmental Engineering
Structural Engineering, Mechanics and Materials

Name ______________________________

MS Comprehensive Examination

Mechanics

Problem 1. (50 points)
Consider an anti-plane problem of linear elastic solid, i.e. the elastic solid has the following displacement field:

$$u(x, y, z) \equiv 0,\ v(x, y, z) \equiv 0, \ \text{ and } \ w = w(x, y) \ \text{ does not depend on z !}$$

(1) Find all the strain components in terms of displacement fields $u = 0, v = 0$ and $w(x, y)$;

(2) Find all the stress components by assuming that the media is a linear elastic solid, and Young's modulus E, shear modulus G and the Poisson's ratio ν are all given;

(3) Consider the three-dimensional equilibrium equations as follows,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + b_x \equiv 0,$$
$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + b_y \equiv 0,$$
$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0 \ ,$$

in which the body forces $b_x = b_y = 0$, and $b_z = b_z(x, y)$. Write the equilibrium equations in terms of displacement field $u = v = 0$ and $w(x, y)$ with given elastic constants and the body forces.

Problem 2. (50 points)
Consider a simply supported beam subjected distributed load $q(x)$ with the following governing equations

$$\frac{d^2}{dx^2}\Big(EI\frac{d^2v}{dx^2}\Big) = q(x), \ \ \forall\, 0 \le x \le L$$

and the boundary conditions,

$$v(0) = 0, \ \ EI\frac{d^2v}{dx^2}\Big|_{x=0} = 0; \ v(L) = 0, \ \ EI\frac{d^2v}{dx^2}_{x=L} = 0;$$

where $EI = const.$
Note that

$$\frac{dv}{dx} = \theta(x), \ EI\frac{d^2v}{dx^2} = M(x), \ \text{ and } \ \frac{d}{dx}\Big(EI\frac{d^2v}{dx}\Big) = -V(x); \ \text{ by definition}$$

(1) Write down the total potential energy of the beam;

(2) Write down the expression of the virtual work principle for this beam;

(3) If $q(x) = q_0 = const.$, find the solution of the beam.

UNIVERSITY OF CALIFORNIA AT BERKELEY
SPRING SEMESTER 2016

Department of Civil and Environmental Engineering
Structural Engineering, Mechanics and Materials

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Comprehensive Examination for Master of Science

Mechanics

Problem 1. (30 points)
Consider a linear elastic medium with the Young's modulus E and Poisson's ratio ν or equivalently given Lamé parameters μ and λ. Assume that the elastic body undergoes to the following displacement,

$$u_1 = \gamma x_2; \;\; u_2 = 0; \;\; \text{and} \;\; u_3 = 0 \, . \tag{1}$$

1. Find the both strain tensor and stress tensor.
2. Explain what kind of deformation this is ?

Problem 2.(40 points)
Consider the following linear elastic strain-stress relation in three-dimensional space,

$$\epsilon_{ij} = \frac{1}{E}\Big[(1+\nu)\sigma_{ij} - \nu\delta_{ij}\sigma_{kk}\Big]$$

where ν is Poisson's ratio and E is Young's modulus.

Write down the stress-strain relations for both the plane stress state as well the plane strain state.

Problem 3.(30 points)
(1) Consider a linear elastic solid being under uniform bi-axial load (plane stress), i.e. $\sigma_{11} = \sigma_{22} = \sigma$. What is the maximum shear strain ?

(2) Consider a linear elastic solid being under uniform triaxial load, i.e. $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma$. What is the maximum shear strain ?

Assume that both Young's modulus E and shear modulus μ (or Poisson's ratio ν) are given.