UNIVERSITY OF CALIFORNIA, BERKELEY
Spring Semester 2021

Dept. of Civil and Environmental Engineering
Structural Engineering, Mechanics and Materials

Name: ....

M.S Comprehensive Examination
Analysis

Note:

1. Dimensions, properties and loading are given in consistent units in all problems.
2. All figures are drawn to scale.
3. Calculations should be shown in detail with all intermediate steps.

Formulas

The deformation $\boldsymbol{v}$-basic force $\boldsymbol{q}$ relation of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{v} = \mathbf{f}\,\boldsymbol{q} + \boldsymbol{v}_0 \quad \text{with} \quad \mathbf{f} = \begin{bmatrix} \dfrac{L}{EA} & 0 & 0 \\ 0 & \dfrac{L}{3EI} & -\dfrac{L}{6EI} \\ 0 & -\dfrac{L}{6EI} & \dfrac{L}{3EI} \end{bmatrix} \qquad \boldsymbol{v}_0 = \begin{pmatrix} \varepsilon_0 L + \mathit{\Delta} L_0 \\ -\dfrac{\kappa_0 L}{2} \\ \dfrac{\kappa_0 L}{2} \end{pmatrix} + \begin{pmatrix} 0 \\ -\dfrac{wL^3}{24EI} \\ \dfrac{wL^3}{24EI} \end{pmatrix}$$

where L is the element length, EA the axial stiffness, EI the flexural stiffness, ε_0 an initial axial strain, $\mathit{\Delta} L_0$ a lack-of-fit, κ_0 a uniform curvature field, and w a uniformly distributed transverse load.

The inverse relation between the basic forces $\boldsymbol{q}$ and the element deformations $\boldsymbol{v}$ of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{q} = \mathbf{k}\boldsymbol{v} + \boldsymbol{q}_0 \quad \text{with} \quad \mathbf{k} = \begin{bmatrix} \dfrac{EA}{L} & 0 & 0 \\ 0 & \dfrac{4EI}{L} & \dfrac{2EI}{L} \\ 0 & \dfrac{2EI}{L} & \dfrac{4EI}{L} \end{bmatrix} \qquad \boldsymbol{q}_0 = \begin{pmatrix} -EA\left(\varepsilon_0 + \frac{\mathit{\Delta} L_0}{L}\right) \\ EI\kappa_0 \\ -EI\kappa_0 \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{wL^2}{12} \\ -\dfrac{wL^2}{12} \end{pmatrix}$$

The symbolic inverse of a 2x2 matrix $\mathbf{M}$ is

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad \rightarrow \quad \mathbf{M}^{-1} = \frac{1}{det(M)} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix} \text{ with } \quad det(M) = M_{11}M_{22} - M_{12}M_{21}$$

1. Problem (50% weight)

The homogeneous, prismatic simply supported girder in Fig. 1 with flexural stiffness EI is subjected to a uniform load w over the left half of its span.

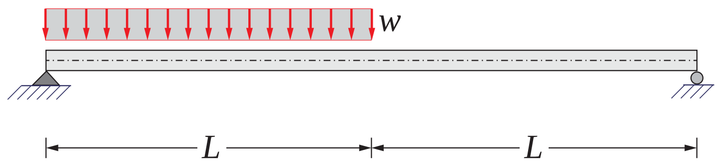


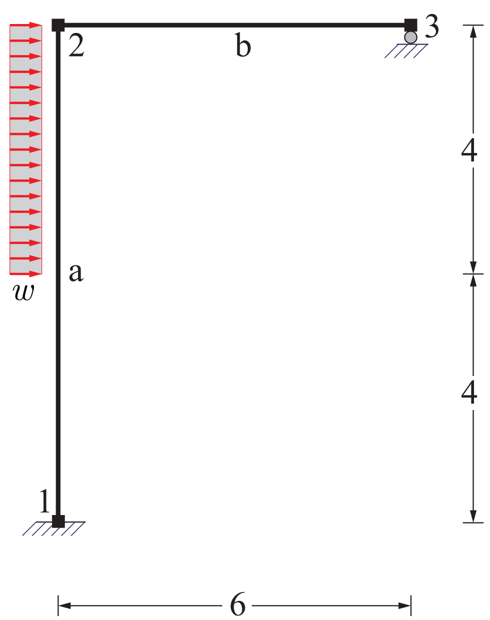
Figure 1: Simply supported girder

You are asked to answer the following questions in terms of w, L, and EI:

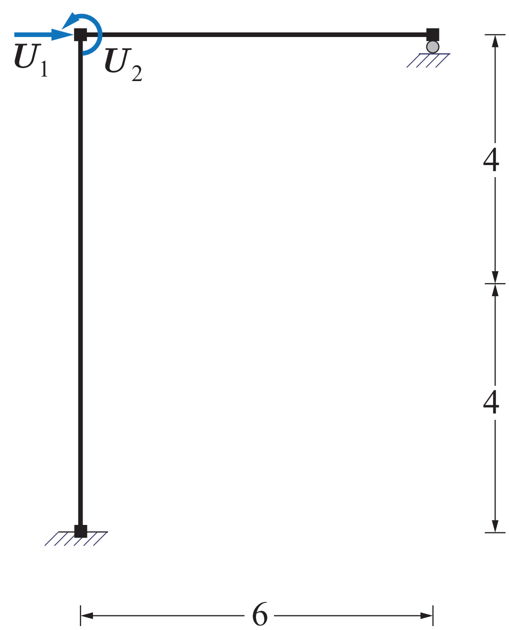
1. Determine the maximum bending moment.
2. Determine the vertical translation at midspan.
3. Determine the end deformations of the girder.

2. Problem (50% weight)

Fig. 2(a) shows the model for a column-girder assembly consisting of the column element a and the girder element b. Under the assumption that both elements are inextensible the model has two independent free degrees-of-freedom (DOFs), as shown in Fig. 2(b). The column has flexural stiffness $EI_c = 80,000$ units and the girder has flexural stiffness $EI_g = 60,000$ units. The model is subjected to a uniform load w of 10 units over the upper half of the column, as Fig. 2(a) shows.



(a) Model geometry and loading



(b) Independent free DOFs

Figure 2: Column-girder model

You are asked to answer the following questions by taking advantage of the results from problem #1:

1. Confirm that the horizontal translation U_1 is equal to $2.6667 \cdot 10^{-2}$ and the rotation U_2 is equal to $-2.3333 \cdot 10^{-3}$ under the given loading.
2. Determine the bending moments and draw the bending moment diagram.
3. Draw the deformed shape of the structural model.

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where L is the element length, EA the axial stiffness, EI the flexural stiffness, ε_0 an initial axial strain, ΔL_0 a lack-of-fit, κ_0 a uniform curvature field, and w a uniformly distributed transverse load.

The inverse relation between the basic forces $\boldsymbol{q}$ and the element deformations $\boldsymbol{v}$ of a homogeneous, prismatic, 2d frame element is:

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1. Problem (50% weight)

The propped cantilever beam of length $2L$ in Fig. 1 consists of two segments of equal length L, one of flexural stiffness $2EI$ and the other of flexural stiffness EI connected with a flexural hinge at midspan. The segment of flexural stiffness EI is subjected to a uniformly distributed load w.

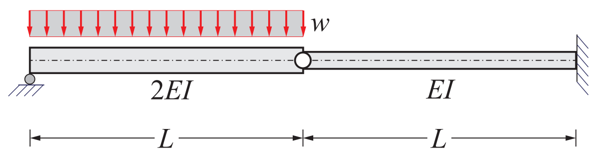

Figure 1: Propped cantilever beam with hinge at midspan

You are asked to answer the following questions:

1. Draw the bending moment diagram making sure to record the end moment values for all elements in the figure after clarifying your sign convention for positive/negative moments.
2. Determine the hinge deformation (rotation) in terms of L, EI and w.

2. Problem (50% weight)

The braced frame in Fig. 2 consists of two *inextensible* frame elements a and b with flexural stiffness EI of 30,000 and a *prestressed* brace element c with axial stiffness EA of 3,000. The initial prestressing force of the brace is 10. The braced frame is subjected to a uniformly distributed load $w = 10$.

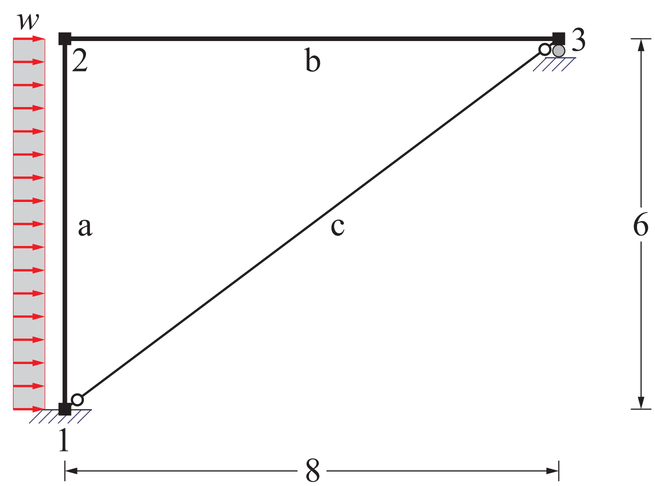

Figure 2: Braced frame under horizontal uniform load w

You are asked to answer the following questions:

1. Draw the bending moment diagram making sure to record the end moment values for all elements in the figure after clarifying your sign convention for positive/negative moments. Determine the largest positive and negative moment value in element a.
2. Determine the support reactions and check global equilibrium.
3. Draw the deformed shape of the braced frame under the uniformly distributed load w.

UNIVERSITY OF CALIFORNIA, BERKELEY
Spring Semester 2018

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3. Calculations should be shown in detail with all intermediate steps; it is recommended to manipulate expressions symbolically as far as possible and substitute numbers only at or near the end.

1. Problem (50% weight)

All elements of the structure in the figure can be assumed inextensible. Under this assumption, there are six basic forces of primary interest. Assume that the corresponding deformations V_1 through V_6 are supplied.

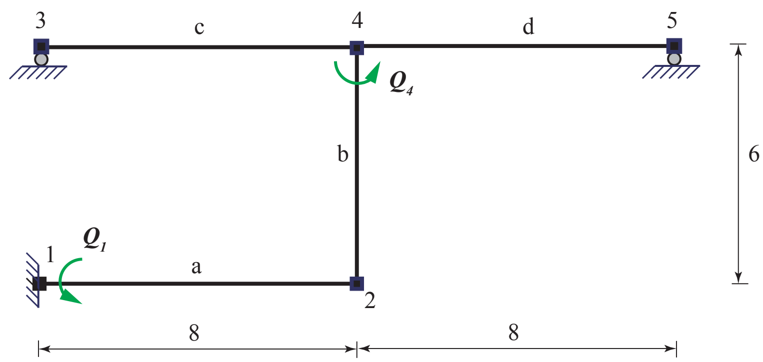

Figure 1: Structural geometry

Determine the symbolically the compatibility conditions (in terms of the deformations V_1 through V_6) using the basic forces Q_1 and Q_4 as redundants, where Q_1 is the flexural basic force at end i of element a and Q_4 is the flexural basic force at end j of element b.

2. Problem (50% weight)

The plastic flexural capacity of elements a through c is 150 units, while the plastic flexural capacity of element d is 200 units. The axial capacity of the frame elements is very large. The axial capacity of the brace element e is 50 units. The plastic hinge distribution at incipient collapse is shown in Fig. 2.

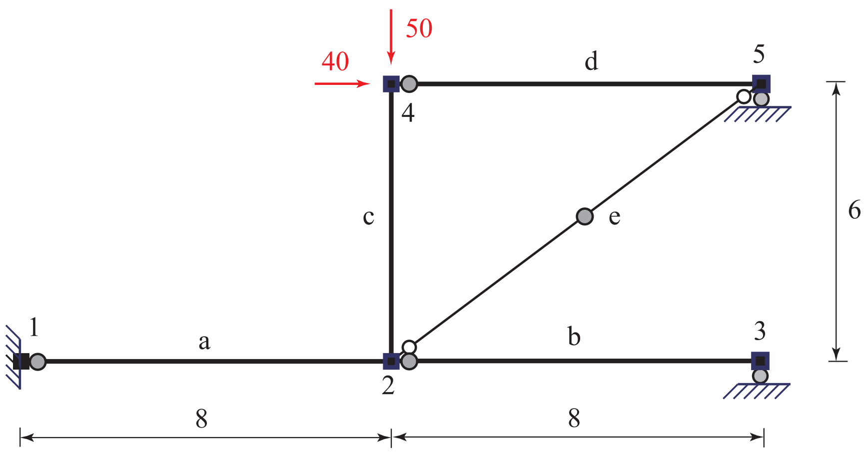

Figure 2: Structure geometry, and loading, and plastic hinge locations

You are asked the following question:

1. Determine the collapse load factor λ_c of the structure under the given loading.

2. Draw the bending moment distribution at incipient collapse indicating clearly which way the frame elements bend.