UNIVERSITY OF CALIFORNIA, BERKELEY
Spring Semester 2021

Dept. of Civil and Environmental Engineering
Structural Engineering, Mechanics and Materials

Name:___________________________________

MS Comprehensive Examination – Dynamics

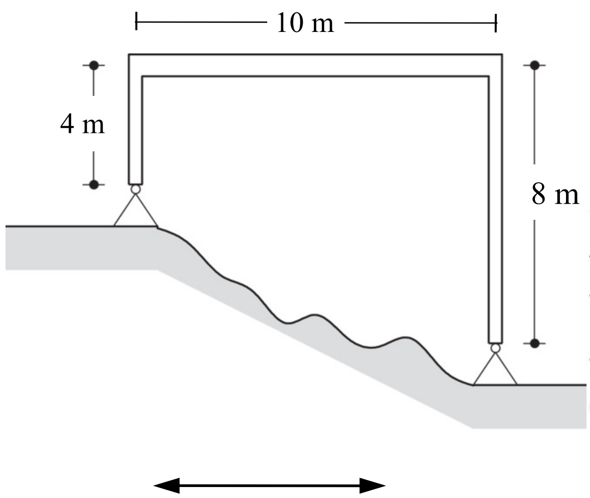
Note: Two pages of useful formulas follow this page.

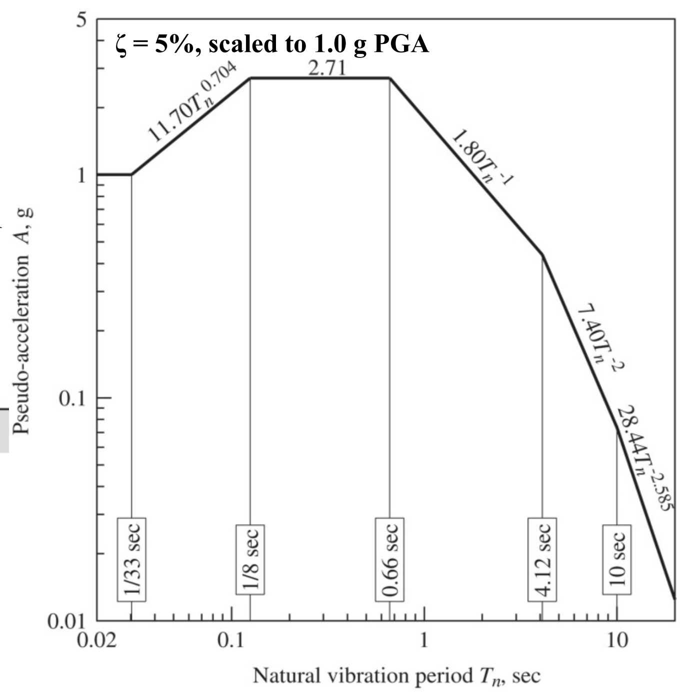
Question 1 (40% weight)

A small one-story reinforced-concrete (E = 20 GPa) building is idealized as a massless frame supporting a 4,000 kg mass at the beam level. Each 25-cm-square column is hinged at the base. Assume that all members are inextensible, and the beam is rigid in flexure. Using the design spectrum shown (5% damping), but scaled to 0.25g PGA,

a) Determine the peak displacement of the structure at the beam level.

b) Draw the bending moment and shear diagrams corresponding to the displacement in a), assuming the structure sways to the right. Consider only earthquake loading, not gravity.





Question 2 (60% weight)

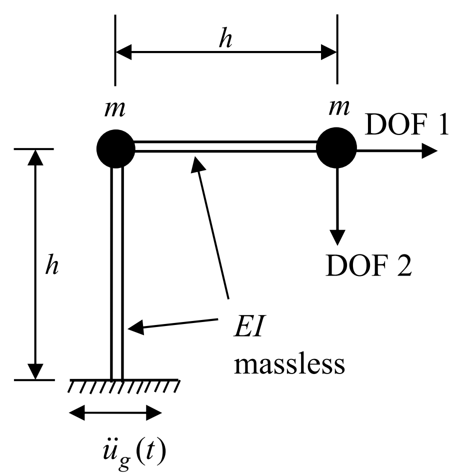
Consider the inverted L-shape structure shown to the right. All members can be considered axially inextensible. For the degrees of freedom shown in the figure, the stiffness matrix is:

$$\mathbf{k} = \begin{bmatrix} 48 & -18 \\ -18 & 12 \end{bmatrix} \frac{EI}{7h^3}$$

a) Determine the matrix equation of motion for the system that is subjected to horizontal ground acceleration $\ddot{u}_g$.

b) Compute the natural frequencies and mode shapes of the structure. Normalize the modes so that $\phi_{1n} = 1$.

c) In the expression $M_b(t) = M_{b1}^{\text{st}} A_1(t) + M_{b2}^{\text{st}} A_2(t)$ that gives the moment at the base of the structure in terms of the modal pseudo-accelerations $A_1(t)$ and $A_2(t)$, determine M_{b1}^{st} and M_{b2}^{st}.



Basic Definitions and Equations

$$\omega_n = \sqrt{\frac{k}{m}} \qquad T_n = \frac{2\pi}{\omega_n} \qquad f_n = \frac{1}{T_n} \qquad (u_{st})_o = \frac{p_o}{k}$$

$$\varsigma = \frac{c}{2m\omega_n} = \frac{c}{c_{cr}}$$

$\varsigma > 1$ Overdamped

$\varsigma = 1$ Critically damped

$\varsigma < 1$ Underdamped

$$\omega_D = \omega_n\sqrt{1-\varsigma^2}$$

Free Vibration $m\ddot{u} + c\dot{u} + ku = 0$

For $\varsigma = 0$: $\quad u(t) = u(0)\cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t$

For $0 < \varsigma < 1$:

$$u(t) = e^{-\varsigma\omega_n t}\left(u(0)\cos\omega_D t + \frac{\dot{u}(0) + \varsigma\omega_n u(0)}{\omega_D}\sin\omega_D t\right)$$

Decay of Motion

$$\frac{u_1}{u_{j+1}} = \exp\left(\frac{2j\pi\varsigma}{\sqrt{1-\varsigma^2}}\right)$$

Free Vibration Test

$$\varsigma = \frac{1}{2\pi j}\ln\frac{u_i}{u_{i+j}}$$

Harmonic Excitation $m\ddot{u} + c\dot{u} + ku = p_o \sin\omega t$

Steady State Response

$$u(t) = u_o \sin(\omega t - \phi)$$

$$R_d = \frac{u_o}{(u_{st})_o} = \frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + [2\zeta(\omega/\omega_n)]^2}}$$

$$\varphi = \tan^{-1}\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}$$

Resonance at $\omega_n\sqrt{1-2\zeta^2}$ with $R_d = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$

Half-Power Bandwidth

$$\frac{\omega_b - \omega_a}{\omega_n} = 2\zeta$$

Vibration Generator: $\quad p(t) = (m_e e\omega^2)\sin\omega t$

Transmissibility $\quad TR = (f_T)_o / p_o = \ddot{u}_o^t / \ddot{u}_{go}$

$$TR = R_d\sqrt{1 + [2\zeta(\omega/\omega_n)]^2}$$

Equivalent Viscous Damping

$$\zeta_{eq} = \frac{1}{4\pi}\frac{E_D}{E_{So}}$$

Arbitrary Excitation $m\ddot{u} + c\dot{u} + ku = p(t)$

Reponse to unit Impulse: $\quad p(t) = \delta(t-\tau)$

$$h(t-\tau) \equiv u(t) = \frac{1}{m\omega_n}\sin(\omega_n(t-\tau)) \qquad \zeta = 0$$

Duhamel's Integral

$$u(t) = \int_0^t p(\tau)h(t-\tau)d\tau$$

Response to Step Force, $\varsigma = 0$

$$u(t) = (u_{st})_o(1-\cos\omega_n t)$$

Response to Ramp Force, $\varsigma = 0$

$$p(t) = p_o\frac{t}{t_r} \qquad u(t) = (u_{st})_o\left(\frac{t}{t_r} - \frac{\sin\omega_n t}{\omega_n t_r}\right)$$

Response to Rectangular Pulse

$$R_d = \begin{cases} 2\sin\pi t_d/T_n & t_d/T_n \le \frac{1}{2} \\ 2 & t_d/T_n \ge \frac{1}{2} \end{cases}$$

Short Pulse $\quad I = \int p(t)dt \qquad u(t) = I\left(\frac{1}{m\omega_n}\sin\omega_n t\right)$

Earthquake Response $p(t) = p_{eff}(t) = -m\ddot{u}_g(t)$

$$u \equiv u(t, T_n, \zeta) \qquad u_o \equiv u_o(T_n, \zeta)$$

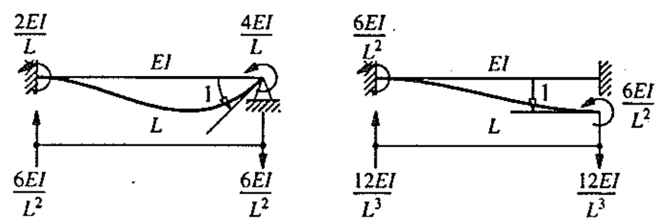
$$D \equiv u_o \qquad V = \omega_n D \qquad A = \omega_n^2 D$$

$$\frac{A}{\omega_n} = V = \omega_n D \qquad E_{so} = \frac{mV^2}{2} \qquad f_{so} = kD = mA$$

For One Story Structure

$$V_{bo} = f_{so} = \frac{A}{g}w \qquad M_{bo} = hV_{bo}$$

Stiffness Coefficients for a Flexural Element



EQ Reponse of Inelastic Systems $m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = p_{eff}(t) = -m\ddot{u}_g(t)$

Normalized Yield Strength

$$\bar{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o}$$

f_y and u_y are yield strength and yield deformation

f_o and u_o are peak force and deformation in corresponding linear system

Yield Strength Reduction Factor

$$R_y = \frac{f_o}{f_y} = \frac{u_o}{u_y} \qquad f_o = ku_o$$

f_o is minimum strength required for structure to remain elastic

Ductility Factor

$$\mu = \frac{u_m}{u_y}$$

u_m is peak deformation of elastoplastic system

Response Spectrum for Inelastic Systems

$$D_y = u_y \qquad V_y = \omega_n D_y \qquad A_y = \omega_n^2 D_y$$

$$f_y = \frac{A_y}{g}w \qquad u_m = \mu\left(\frac{T_n}{2\pi}\right)^2 A_y$$

Generalized SDOF Systems: Distributed Mass and Elasticity

For Assumed Shape Function $\psi(x)$

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx \qquad \tilde{\Gamma} = \frac{\tilde{L}}{\tilde{m}} \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{L} = \int_0^L m(x)\psi(x)dx \qquad z_o = \tilde{\Gamma}D$$

At Height x Above the Base

$$u_o(x) = \tilde{\Gamma}D\psi(x) \qquad f_o(x) = \tilde{\Gamma}m(x)\psi(x)A$$

Static Analysis of the tower due to $f_o(x)$ provides interal forces.

Base Shear and Moment:

$$V_{bo} = V_o(0) = \tilde{L}\tilde{\Gamma}A \qquad M_{bo} = M_o(0) = \tilde{L}^\theta\tilde{\Gamma}A$$

$$\tilde{L}^\theta = \int_0^L x m(x)\psi(x)dx$$

Equation of Motion, MDOF $\mathbf{m}\ddot{\mathbf{u}}+\mathbf{c}\dot{\mathbf{u}}+\mathbf{k}\mathbf{u}=\mathbf{p}(t)$

Earthquake Excitation

$$\mathbf{p}(t)=\mathbf{p}_{eff}(t)=-\mathbf{m}\,\iota\,\ddot{u}_g(t)$$

where $\iota \equiv$ influence vector

Static Condesation

$\mathbf{u}_t \equiv$ translational DOF $\mathbf{u}_o \equiv$ rotational DOF

$$\begin{bmatrix}\mathbf{m}_{tt} & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{u}}_t\\ \ddot{\mathbf{u}}_o\end{Bmatrix}+\begin{bmatrix}\mathbf{k}_{tt} & \mathbf{k}_{to}\\ \mathbf{k}_{ot} & \mathbf{k}_{oo}\end{bmatrix}\begin{Bmatrix}\mathbf{u}_t\\ \mathbf{u}_o\end{Bmatrix}=\begin{Bmatrix}\mathbf{p}_t(t)\\ \mathbf{0}\end{Bmatrix}$$

$$\hat{\mathbf{k}}_{tt}=\mathbf{k}_{tt}-\mathbf{k}_{ot}^T\mathbf{k}_{oo}^{-1}\mathbf{k}_{ot}$$

To find $\mathbf{u}_t(t)$ solve: $\mathbf{m}_{tt}\ddot{\mathbf{u}}_t+\hat{\mathbf{k}}_{tt}\mathbf{u}_t=\mathbf{p}_t(t)$

To find $\mathbf{u}_o(t)$ use: $\mathbf{u}_o=-\mathbf{k}_{oo}^{-1}\mathbf{k}_{ot}\mathbf{u}_t$

Natural Frequencies and Modes

To find ω_n^2, use characteristic equation: $\det\left[\mathbf{k}-\omega_n^2\mathbf{m}\right]=0$

To find ϕ_n, solve: $\left[\mathbf{k}-\omega_n^2\mathbf{m}\right]\phi_n=0$

Orthogonality of Modes

$$\phi_n^T\mathbf{k}\phi_r=0 \qquad \phi_n^T\mathbf{m}\phi_r=0$$

Modal Expansion of Displacements

$$\mathbf{u}=\sum_{r=1}^{N}\phi_r q_r=\Phi\mathbf{q} \qquad \text{where: } q_n=\frac{\phi_n^T\mathbf{m}\mathbf{u}}{M_n}$$

For Initial Conditions: $\mathbf{u}(0),\dot{\mathbf{u}}(0)$

$$q_n(0)=\frac{\phi_n^T\mathbf{m}\mathbf{u}(0)}{M_n} \qquad \dot{q}_n(0)=\frac{\phi_n^T\mathbf{m}\dot{\mathbf{u}}(0)}{M_n}$$

Modal Equations

$$M_n\ddot{q}_n+C_n\dot{q}_n+K_nq_n=P_n(t)$$

$$\ddot{q}_n+2\zeta_n\omega_n\dot{q}_n+\omega_n^2q_n=\frac{P_n(t)}{M_n}$$

$$K_n=\phi_n^T\mathbf{k}\phi_n \qquad M_n=\phi_n^T\mathbf{m}\phi_n \qquad P_n(t)=\phi_n^T\mathbf{p}(t)$$

For classical damping: $C_n=\phi_n^T\mathbf{c}\phi_n$ $\qquad \varsigma_n=\dfrac{C_n}{2M_n\omega_n}$

Dynamic Response

Solve modal equations for $q_n(t)$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\mathbf{u}_n(t)=\sum_{n=1}^{N}\boldsymbol{\varphi}_nq_n(t)$$

Equivalent static forces, nth mode

$$\mathbf{f}_n(t)=\omega_n^2\mathbf{m}\boldsymbol{\varphi}_nq_n(t)$$

Internal forces by static analysis

EQ Response History Analysis $\mathbf{m}\ddot{\mathbf{u}}+\mathbf{c}\dot{\mathbf{u}}+\mathbf{k}\mathbf{u}=\mathbf{p}_{eff}(t)=-\mathbf{m}\iota\ddot{u}_g(t)$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\phi_nq_n(t) \qquad \mathbf{m}\iota=\sum_{n=1}^{N}s_n=\sum_{n=1}^{N}\Gamma_n\mathbf{m}\phi_n$$

$$\Gamma_n=\frac{L_n}{M_n} \qquad L_n=\phi_n^T\mathbf{m}\iota \qquad M_n=\phi_n^T\mathbf{m}\phi_n$$

$$q_n(t)=\Gamma_nD_n(t)$$

Modal Equations for Classically-Damped Systems

$$\ddot{q}_n+2\varsigma_n\omega_n\dot{q}_n+\omega_n^2q_n=-\Gamma_n\ddot{u}_g(t)$$

Modal Response

$$\mathbf{u}_n(t)=\phi_nq_n(t)=\Gamma_n\phi_nD_n(t)$$

$$\mathbf{f}_n(t)=\mathbf{s}_nA_n(t) \qquad A_n(t)=\omega_n^2D_n(t)$$

$$r_n(t)=r_n^{st}A_n(t)$$

Modal Static Response

Forces: by statics

Displacements: $\mathbf{u}_n^{st}=\dfrac{\Gamma_n}{\omega_n^2}\phi_n$

Total Response

$$r(t)=\sum_{n=1}^{N}r_n(t)=\sum_{n=1}^{N}r_n^{st}A_n(t)$$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\mathbf{u}_n(t)=\sum_{n=1}^{N}\Gamma_n\phi_nD_n(t)$$

For Multistory Building with Symmetric Plan

$$\Gamma_n=\frac{L_n^h}{M_n} \qquad L_n^h=\sum_{j=1}^{N}m_j\phi_{jn} \qquad M_n=\sum m_j\phi_{jn}^2$$

$$\mathbf{s}_n=\Gamma_n\mathbf{m}\phi_n \qquad s_{jn}=\Gamma_nm_j\phi_{jn}$$

Effective Modal Mass and Height

$$M_n^*=\Gamma_nL_n^h=\frac{\left(L_n^h\right)^2}{M_n} \qquad h_n^*=\frac{L_n^\theta}{L_n^h} \qquad L_n^\theta=\sum_{j=1}^{N}h_jm_j\phi_{jn}$$

$$\sum_{n=1}^{N}M_n^*=\sum_{j=1}^{N}m_j \qquad \sum_{n=1}^{N}h_n^*M_n^*=\sum_{j=1}^{N}h_jm_j$$

Total Response

$$r(t)=\sum_{n=1}^{N}r_n(t)=\sum_{n=1}^{N}r_n^{st}A_n(t)$$

Earthquake Response Spectrum Analysis

$$r_{no}=r_n^{st}A_n \qquad r_o\equiv\max_t|r(t)| \qquad r_o\cong\left(\sum_{n=1}^{N}r_{no}^2\right)^{1/2}$$

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Structural Engineering, Mechanics & Materials
Spring Semester, 2020

Comprehensive Examination - Dynamics

Problem 1 (50% weight)
A single-degree-of-freedom oscillator has a mass of 100 lb, a natural period of 0.5 seconds, and a fraction of critical damping of 5%. The oscillator is subjected to a harmonic ground motion with an amplitude of 0.1g and a frequency of 4 Hz.

Determine the maximum total displacement of the mass.

Problem 2 (50% weight)
Figure 1 shows a one-story portal frame. Assume the floor remains horizontal during motion. The floor weight is $W = 80$ kips. The columns have a height of $h = 12$ ft and a flexural rigidity of $EI = 2 \times 10^6$ lb-ft^2. Assume that $\zeta = 5\%$, the force–deformation relation is elastoplastic, the design earthquake has a peak ground acceleration of 0.6g, and the elastic design spectrum in Fig. 6.9.5 applies (after scaling to the correct PGA).

Determine the lateral force for which the frame should be designed if:

(a) the system is required to remain elastic
(b) the allowable ductility factor is 4.

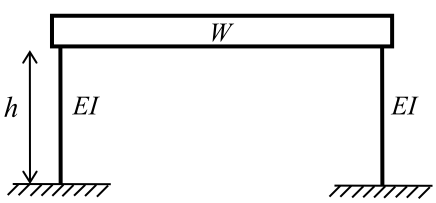

Figure 1

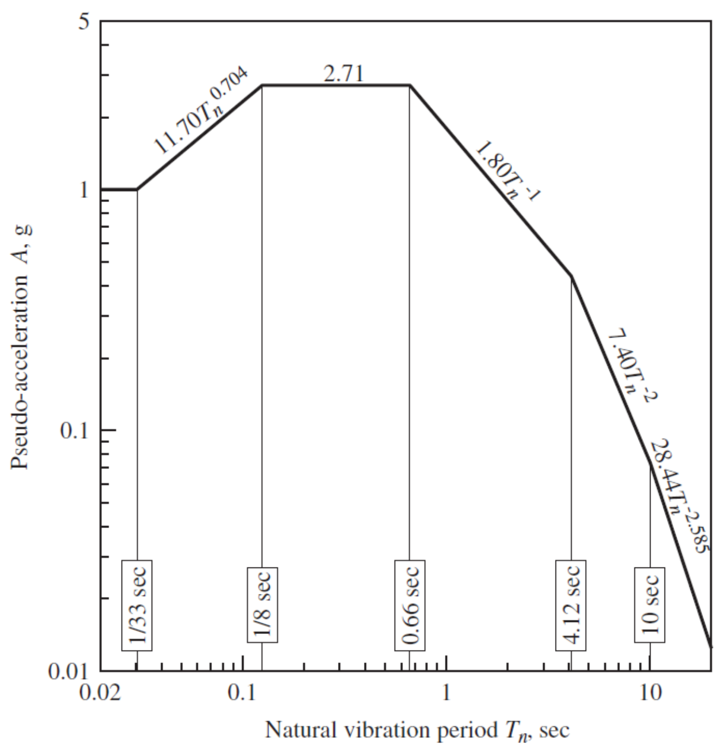

Figure 6.9.5 Elastic pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1$g, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 5\%$.

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Structural Engineering, Mechanics & Materials
Spring Semester, 2019

Comprehensive Examination - Dynamics

Problem 1 (30% weight)
Consider an industrial machine that weighs 1000 lbs and is supported on spring-type isolators of total stiffness $k = 3000$ lb/ft. The machine operates at a frequency of $f = 2$ Hz with a force unbalance of $p_o = 100$ lbs. Assume 5% damping.

(a) Determine the maximum displacement of the machine (from the at rest position) during steady state oscillation.

Problem 2 (30% weight)
The elevated water tank of Figure 1a weighs 90 kips. The tower has a lateral stiffness of 10 kips/in, and is subjected to the time-varying force $p(t)$ shown in Figure 1b. Assume zero damping and that the tower is at rest at time $t = 0$. Treating the water tower as an SDOF system, determine:

(a) the equation that describes the dynamic response $u(t)$ for $0 < t < 4$ seconds.
(b) the equation that describes the dynamic response $u(t)$ for $t > 4$ seconds.

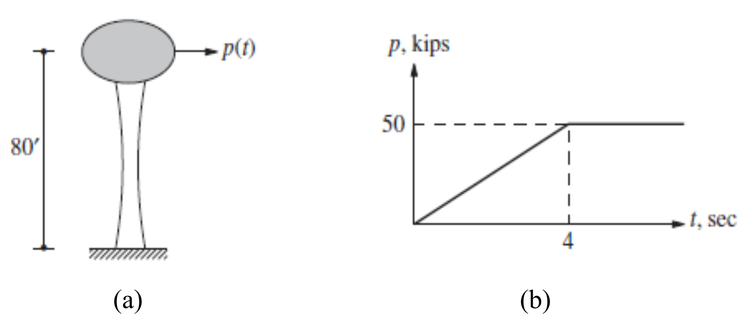


Figure 1

<u>Problem 3</u> (40% weight)

Figure 2 shows a two-storey sway-frame. Assume the floors remain horizontal during motion. The weight of each storey is $W = 100$ kips. All columns have a height of $h = 12$ ft and a flexural rigidity of $EI = 5 \times 10^6$ lb-ft^2.

(a) Assuming zero damping, write the equation of motion.

(b) Assuming zero damping, the natural vibration periods are 2.15 seconds and 0.82 seconds. Determine the mode shapes.

(c) Assuming 2% damping, use the response spectra in Figure 3 to estimate the maximum acceleration of the top storey.

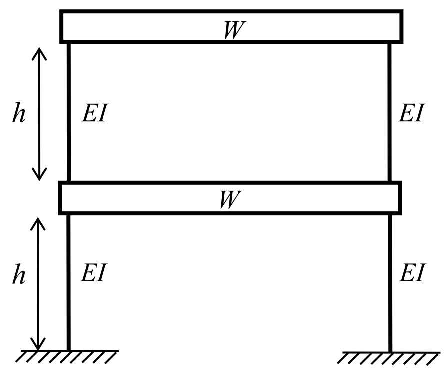
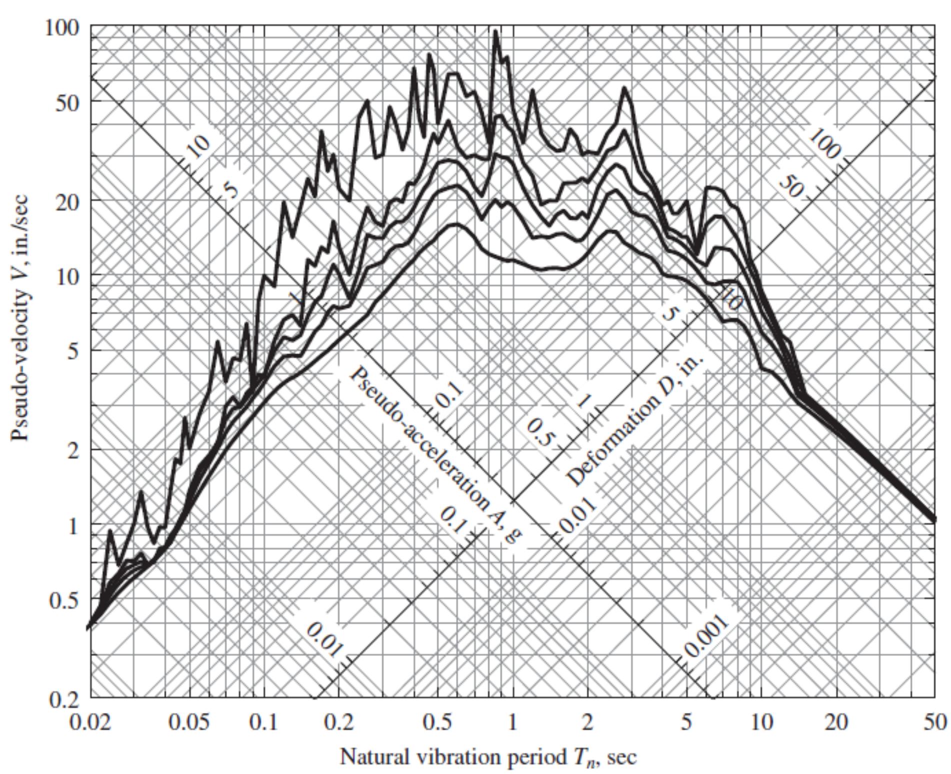


Figure 2



* Spectra shown for $\zeta = 0, 2, 5, 10, 20\%$

Figure 3