UNIVERSITY OF CALIFORNIA, BERKELEY
Fall Semester 2021

Dept. of Civil and Environmental Engineering
Structural Engineering, Mechanics and Materials

Name: ....

Ph.D. Preliminary Examination
Analysis

Note:

1. Dimensions, properties and loading are given in consistent units in all problems.
2. All figures are drawn to scale.

Formulas

The deformation $\boldsymbol{v}$-basic force $\boldsymbol{q}$ relation of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{v} = \mathbf{f}\,\boldsymbol{q} + \boldsymbol{v}_0 \quad \text{with} \quad \mathbf{f} = \begin{bmatrix} \dfrac{L}{EA} & 0 & 0 \\ 0 & \dfrac{L}{3EI} & -\dfrac{L}{6EI} \\ 0 & -\dfrac{L}{6EI} & \dfrac{L}{3EI} \end{bmatrix} \qquad \boldsymbol{v}_0 = \begin{pmatrix} \varepsilon_0 L + \Delta L_0 \\ -\dfrac{\kappa_0 L}{2} \\ \dfrac{\kappa_0 L}{2} \end{pmatrix} + \begin{pmatrix} 0 \\ -\dfrac{wL^3}{24EI} \\ \dfrac{wL^3}{24EI} \end{pmatrix}$$

where L is the element length, EA the axial stiffness, EI the flexural stiffness, ε_0 an initial axial strain, ΔL_0 a lack-of-fit, κ_0 a uniform curvature field, and w a uniformly distributed transverse load.

The inverse relation between the basic forces $\boldsymbol{q}$ and the element deformations $\boldsymbol{v}$ of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{q} = \mathbf{k}\,\boldsymbol{v} + \boldsymbol{q}_0 \quad \text{with} \quad \mathbf{k} = \begin{bmatrix} \dfrac{EA}{L} & 0 & 0 \\ 0 & \dfrac{4EI}{L} & \dfrac{2EI}{L} \\ 0 & \dfrac{2EI}{L} & \dfrac{4EI}{L} \end{bmatrix} \qquad \boldsymbol{q}_0 = \begin{pmatrix} -EA\left(\varepsilon_0 + \frac{\Delta L_0}{L}\right) \\ EI\kappa_0 \\ -EI\kappa_0 \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{wL^2}{12} \\ -\dfrac{wL^2}{12} \end{pmatrix}$$

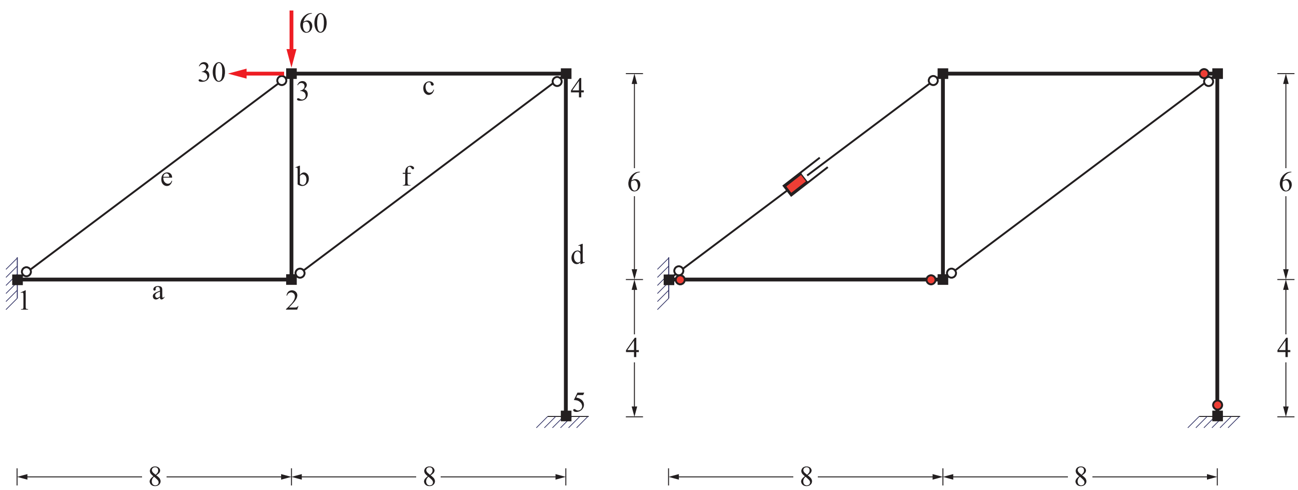
The symbolic inverse of a 2x2 matrix $\mathbf{M}$ is

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad \rightarrow \quad \mathbf{M}^{-1} = \frac{1}{det(M)} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix} \text{ with } \quad det(M) = M_{11}M_{22} - M_{12}M_{21}$$

1. Problem (50% weight)

The braced frame in Fig. 1(a) is subjected to a horizontal force of 30 units and a downward force of 60 units at node 3. The horizontal frame elements a and c have plastic flexural capacity M_p of 200 units, while the vertical frame elements b and d have plastic flexural capacity M_p of 250 units. The frame elements have very high plastic axial capacity N_p. *It is assumed that the presence of an axial force in a frame element does not affect its plastic flexural capacity.* The brace elements e and f have plastic axial capacity N_p of 20 units.

Fig. 1(b) shows the location of the plastic hinges at incipient collapse with a red (gray) filled circle for the frame elements and a red (gray) dashpot for the braces. Specifically, these are located at both ends of element a, at the right end (end j) of element c, at the base (end j) of element d, and in the brace element e.



(a) Geometry and loading

(b) Plastic hinges at incipient collapse

Figure 1: Braced frame geometry and loading and plastic hinge locations at incipient collapse

You are asked to answer the following questions:

1. Determine the collapse load factor λ_c of the structural model under the given reference nodal forces with the upper bound theorem of plastic analysis.
2. Confirm that there is a set of basic element forces that satisfy the equilibrium equations under the nodal forces factored with λ_c without exceeding the plastic capacities of the elements.
3. Draw the bending moment distribution for the set of basic element forces from the preceding question.

2. Problem (50% weight)

The braced frame in Fig. 2(a) consists of 3 *inextensible* frame elements a, b, and c with flexural stiffness $EI = 80,000$ units and two brace elements d and e with axial stiffness EA whose value is missing.

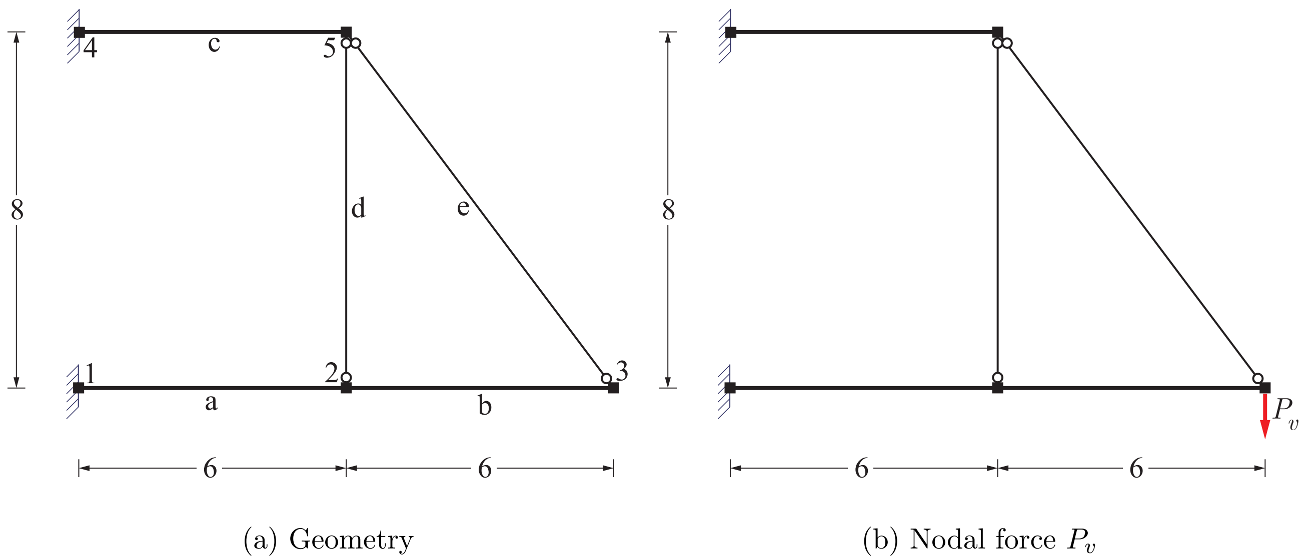


Figure 2: Braced frame under the action of nodal force P_v

Part A

You are asked to answer the following question:

1. Determine the value of the axial stiffness EA for the case that a downward vertical force P_v of 10 units at node 3 shown in Fig. 2(b) gives rise to an axial force of 13.373 units (tension) in element e and to an axial force of -5.7141 units (compression) in element d.

Part B

The model geometry and properties are the same as for Part A. Regardless of your answer for Part A assume that the axial stiffness of the brace elements d and e is $EA = 50,000$ units.
The brace frame is subjected to a downward vertical force of 40 units at node 2 and a downward vertical force of 20 units at node 3, as Fig. 3 shows. *In addition, the braces are prestressed but the value of the initial prestressing force in each is missing.*

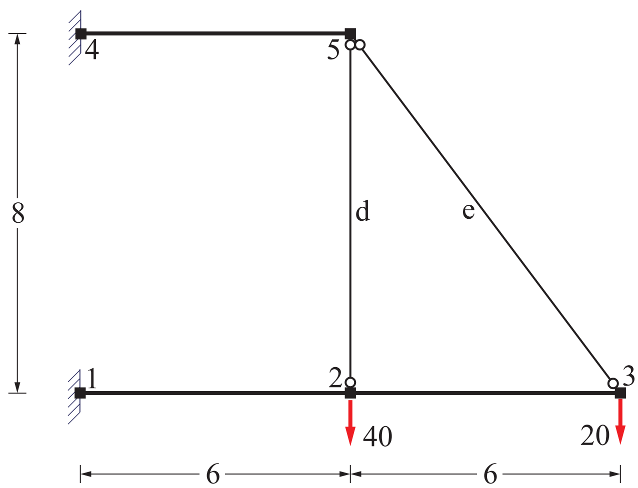


Figure 3: Braced frame with nodal forces and initial prestressing in the braces

You are asked to answer the following questions:

1. Determine the value of the initial prestressing force in the brace element d and the initial prestressing force in the brace element e from the knowledge that the nodal forces in Fig. 3 give rise to an axial force of 42.364 units (tension) in element d and to an axial force of -9.8763 units (compression) in element e.
2. Determine the vertical translation at node 3 under the given loading.

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where L is the element length, EA the axial stiffness, EI the flexural stiffness, ε_0 an initial axial strain, ΔL_0 a lack-of-fit, κ_0 a uniform curvature field, and w a uniformly distributed transverse load.

The inverse relation between the basic forces $\boldsymbol{q}$ and the element deformations $\boldsymbol{v}$ of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{q} = \mathbf{k}\,\boldsymbol{v} + \boldsymbol{q}_0 \quad \text{with} \quad \mathbf{k} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & \frac{2EI}{L} \\ 0 & \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \qquad \boldsymbol{q}_0 = \begin{pmatrix} -EA\left(\varepsilon_0 + \frac{\Delta L_0}{L}\right) \\ EI\kappa_0 \\ -EI\kappa_0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{wL^2}{12} \\ -\frac{wL^2}{12} \end{pmatrix}$$

The symbolic inverse of a 2x2 matrix $\mathbf{M}$ is

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1. Problem (50% weight)

The structural model in Fig. 1 is subjected to a horizontal force of 20 units at node 3 and a downward force of 40 units at node 5. The frame elements a, c and d have plastic flexural capacity M_p of 200 units, while the frame element b has plastic flexural capacity M_p of 240 units. The frame elements have very high plastic axial capacity N_p. *It is assumed that the presence of an axial force in a frame element does not affect its plastic flexural capacity.*

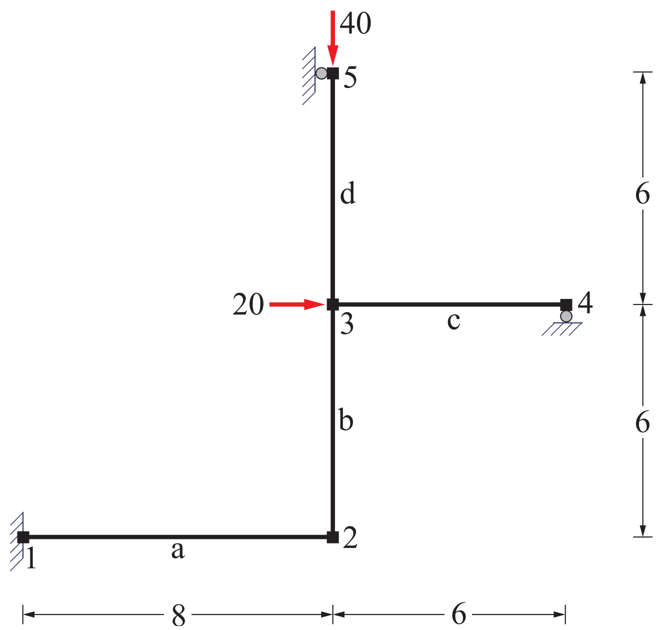

Figure 1

You are asked to answer the following questions:

1. Determine the collapse load factor λ_c of the structural model under the given reference nodal forces.
2. Draw the bending moment distribution at incipient collapse and supply the end moment values for the frame elements.

2. Problem (50% weight)

You are asked to establish *the stiffness matrix* **K** *or the flexibility matrix* **F** for the two vertical translation DOFs of the propped cantilever with overhang in Fig. 2 to serve for the investigation of the vibration properties of the structure with two lumped masses m. The propped cantilever has uniform flexural stiffness EI.

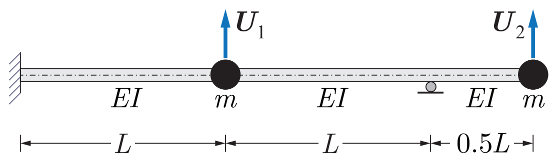

Figure 2: Propped cantilever with overhang

In the derivation of the stiffness or flexibility matrix for the two vertical translation DOFs you may find the relations in Fig. 3 between an applied vertical force P_v or an applied moment M and the resulting bending moment at the root of the propped cantilever helpful.

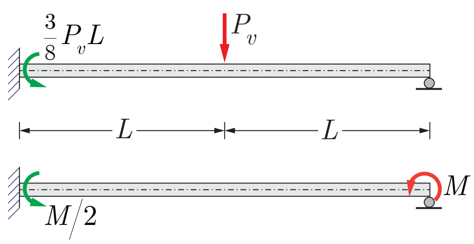

Figure 3: Relations between P_v and M and the bending moment at the root of the propped cantilever

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where L is the element length, EA the axial stiffness, EI the flexural stiffness, ε_0 an initial axial strain, $\mathit{\Delta} L_0$ a lack-of-fit, κ_0 a uniform curvature field, and w a uniformly distributed transverse load.

The inverse relation between the basic forces $\boldsymbol{q}$ and the element deformations $\boldsymbol{v}$ of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{q} = \mathbf{k}\boldsymbol{v} + \boldsymbol{q}_0 \quad \text{with} \quad \mathbf{k} = \begin{bmatrix} \dfrac{EA}{L} & 0 & 0 \\ 0 & \dfrac{4EI}{L} & \dfrac{2EI}{L} \\ 0 & \dfrac{2EI}{L} & \dfrac{4EI}{L} \end{bmatrix} \qquad \boldsymbol{q}_0 = \begin{pmatrix} -EA\left(\varepsilon_0 + \frac{\mathit{\Delta} L_0}{L}\right) \\ EI\kappa_0 \\ -EI\kappa_0 \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{wL^2}{12} \\ -\dfrac{wL^2}{12} \end{pmatrix}$$

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1. Problem (50% weight)

The tied frame in Fig. 1 consists of an *inextensible* beam with flexural stiffness EI, two *inextensible* columns with flexural stiffness EI, and a tie (truss) with axial stiffness EA.

- Under the application of a horizontal force $P_h = 10$ the axial force in the tie is 4.8544 units.
- Under the application of a vertical force $P_v = 20$ the axial force in the tie is 1.4563 units.

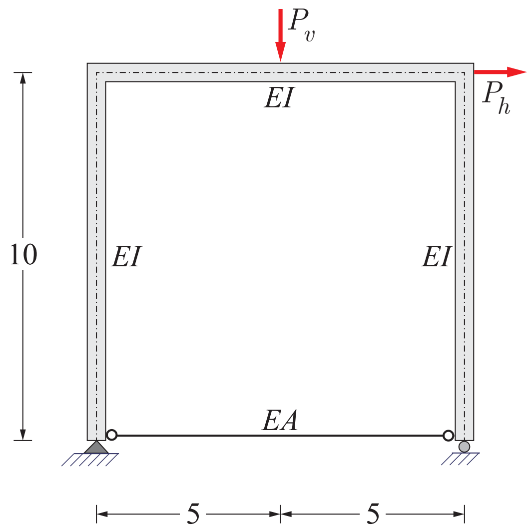

Figure 1: Tied frame

You are asked to answer the following questions:

1. Determine the axial stiffness EA of the tie and the flexural stiffness EI of the beam and columns.
2. Discuss whether there are values of the axial stiffness EA that result in the same bending moment value in absolute terms at the left and at the right beam-column joints under the horizontal force P_h and under the vertical force P_v as separate load cases.

2. Problem (50% weight)

The structural model of a cable-stayed bridge in Fig. 2 consists of three *inextensible* frame elements a, b, and c, and the truss element d.

The frame elements have linear elastic, perfectly-plastic flexural response with flexural stiffness EI of 200,000 units and plastic flexural capacity M_p of 200 units. The frame elements have very high plastic axial capacity N_p. *It is assumed that the presence of an axial force in a frame element does not affect its plastic flexural capacity.*

The truss element has linear elastic, perfectly-plastic axial response with axial stiffness EA of 40,000 units and plastic axial capacity N_p of 40 units.

The structure is subjected to a downward nodal force of 40 units at node 2, as shown in Fig. 2.

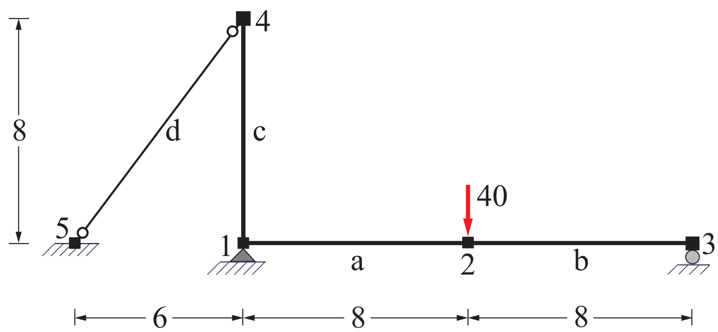

Figure 2: Cable-stayed bridge

You are asked to answer the following questions:

1. Determine the collapse load factor λ_c.
2. Draw the bending moment distribution $M(x)$ at incipient collapse.
3. What is the change of the collapse load factor λ_c, if the truss element d is prestressed with a force of 40 units?
4. What is the required prestressing force so that the incipient collapse state is reached in a single event, i.e. that all hinges necessary for the formation of the collapse mechanism form at the same time?

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3. Degrees of freedom (DOFs) should be numbered starting from the node with lowest number and proceeding in ascending node order.
4. The end i of an element corresponds to the node with lower number and the end j to the node with higher number.

The deformation $\boldsymbol{v}$-basic force $\boldsymbol{q}$ relation of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{v} = \mathbf{f}\,\boldsymbol{q} + \boldsymbol{v}_0 \quad \text{with} \quad \mathbf{f} = \begin{bmatrix} \dfrac{L}{EA} & 0 & 0 \\ 0 & \dfrac{L}{3EI} & -\dfrac{L}{6EI} \\ 0 & -\dfrac{L}{6EI} & \dfrac{L}{3EI} \end{bmatrix} \qquad \boldsymbol{v}_0 = \begin{pmatrix} \varepsilon_0 L + \Delta L_0 \\ -\dfrac{\kappa_0 L}{2} \\ \dfrac{\kappa_0 L}{2} \end{pmatrix} + \begin{pmatrix} 0 \\ -\dfrac{wL^3}{24EI} \\ \dfrac{wL^3}{24EI} \end{pmatrix}$$

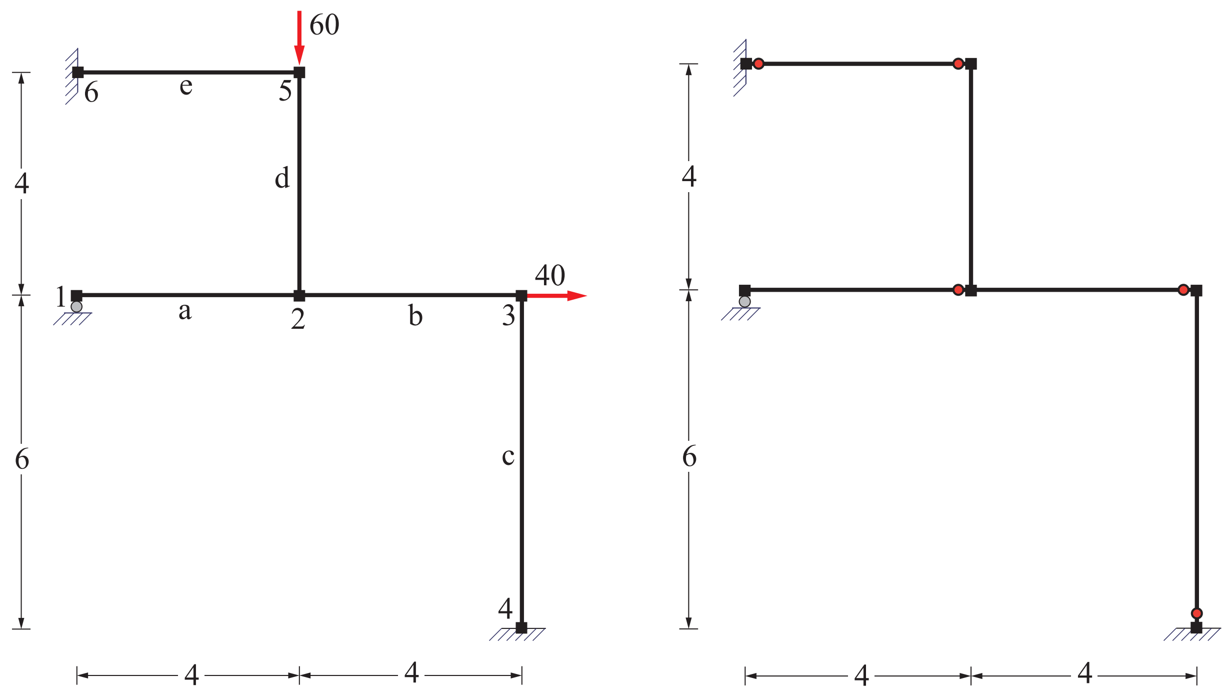
where L is the element length, EA the axial stiffness, EI the flexural stiffness, ε_0 an initial axial strain, ΔL_0 a lack-of-fit, κ_0 a uniform curvature field, and w a uniformly distributed transverse load.

The inverse relation between the basic forces $\boldsymbol{q}$ and the element deformations $\boldsymbol{v}$ of a homogeneous, prismatic, 2d frame element is:

$$\boldsymbol{q} = \mathbf{k}\,\boldsymbol{v} + \boldsymbol{q}_0 \quad \text{with} \quad \mathbf{k} = \begin{bmatrix} \dfrac{EA}{L} & 0 & 0 \\ 0 & \dfrac{4EI}{L} & \dfrac{2EI}{L} \\ 0 & \dfrac{2EI}{L} & \dfrac{4EI}{L} \end{bmatrix} \qquad \boldsymbol{q}_0 = \begin{pmatrix} -EA\varepsilon_0 \\ EI\kappa_0 \\ -EI\kappa_0 \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{wL^2}{12} \\ -\dfrac{wL^2}{12} \end{pmatrix}$$

1. Problem (40% weight)

The structural model of a two story frame in Fig. 1(a) is subjected to a horizontal force of 40 units at node 3 and a vertical force of 60 units at node 4. The horizontal frame elements a, b and e have a perfectly plastic flexural capacity of $M_p = 100$ units, while the vertical elements c and d have a perfectly plastic flexural capacity of $M_p = 120$ units. All frame elements have very large axial capacity. Moreover, the effect of the axial force on the flexural capacity of an element can be neglected.
Fig. 1(b) gives the plastic hinge locations at incipient collapse: they arise at end j of elements a, b and c and at both ends of element e.

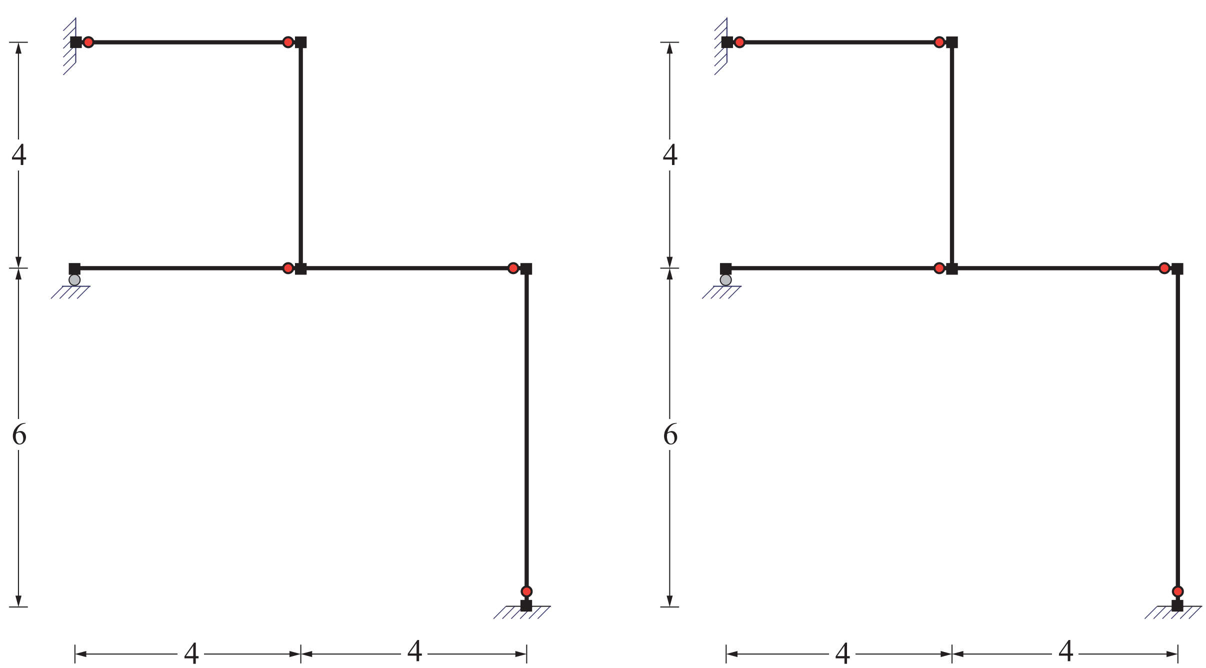


(a) Geometry and loading (b) Plastic hinges at incipient collapse

Figure 1: Two Story Frame

You are asked to answer the following questions:

1. Determine the collapse load factor $\lambda_c \geq 0$ of the frame for the given nodal forces.
2. Draw the collapse mechanism in Fig. 2(a).
3. Confirm the given plastic hinge locations.
4. Draw the bending moment distribution at incipient collapse in Fig. 2(b). Be sure to record the end moment values for all elements in the figure after clarifying your sign convention for positive/negative moments.



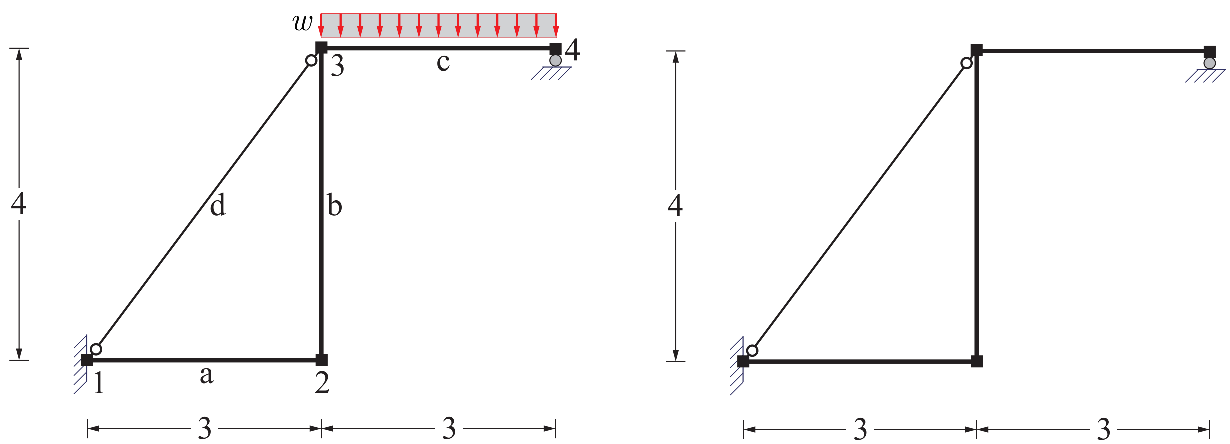
(a) Collapse mechanism

(b) Bending moments at incipient collapse

Figure 2

2. Problem (20% weight)

The braced frame in Fig. 3(a) consists of the *inextensible* frame elements a, b and c and the truss (brace) element d. In the general case, the flexural stiffness EI of each frame element may be different, so that EI_a, EI_b and EI_c are not the same. The truss element has axial stiffness EA.
The braced frame is subjected to a uniformly distributed load w in element c.



(a) Geometry and loading

(b) Independent free DOFs

Figure 3: Braced frame

You are asked to answer the following questions:

1. Identify the independent free degrees of freedom (DOFs) in Fig. 3(b).
2. Set up symbolically the stiffness matrix $\mathbf{K}_f$ and the initial force vector $\boldsymbol{P}_0$ of the braced frame under the given loading.

Recall that the equilibrium equations for the displacement method of analysis take the form

$$\boldsymbol{P}_f = \mathbf{K}_f \boldsymbol{U}_f + \boldsymbol{P}_0$$

where $\boldsymbol{P}_f$ are the applied nodal forces and $\boldsymbol{U}_f$ are the independent free DOF displacements.

3. Problem (40% weight)

The cable-stayed structural model in Fig. 4 consists of the *inextensible* frame elements a, b and c and the *prestressed* cable-stay element d. The flexural stiffness EI of elements a, b and c is 10,000 units and the axial stiffness EA of the cable-stay is 5,000 units. The structure is subjected to a uniformly distributed load $w = 10$ in element a.
The structural analysis of the *prestressed* structure under w gives an axial force of 11.95 units in the cable-stay and a basic flexural force $q_j = -20.66$ for element b (bending moment at the fixed-support).

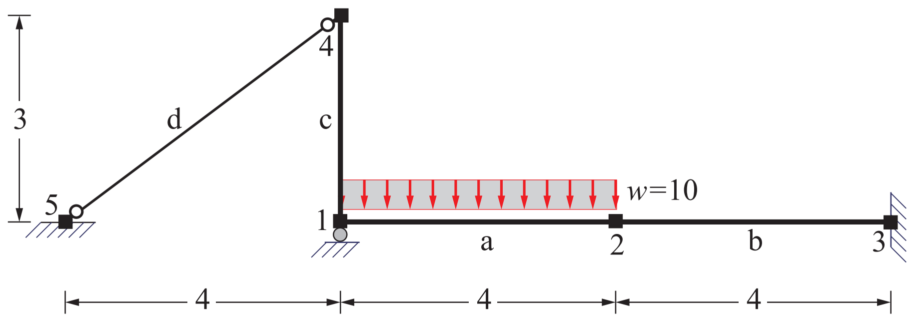


Figure 4: Prestressed cable-stay structure under uniform element load in element a

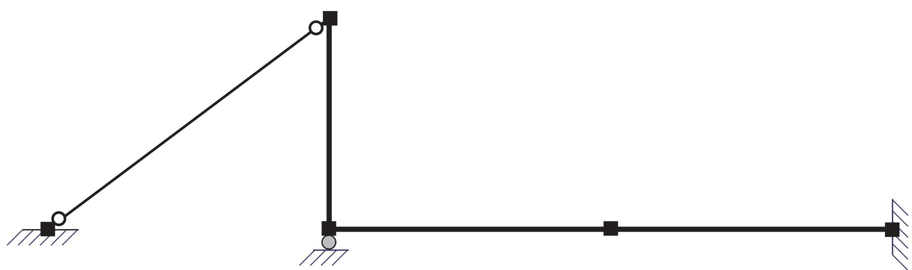


Figure 5: Bending moment distribution

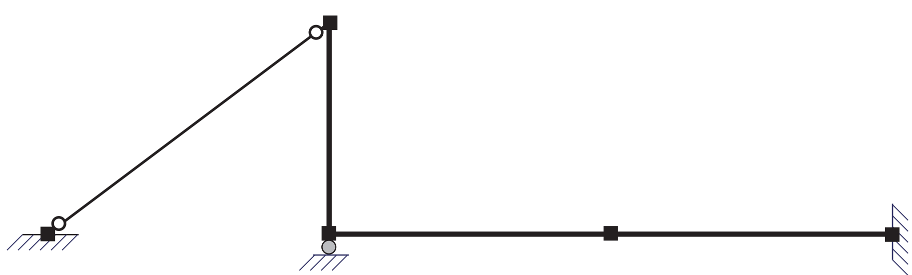


Figure 6: Deformed shape

You are asked to answer the following questions:

1. Draw the bending moment distribution for the *prestressed* structure under w in Fig. 5 after clarifying your sign convention for positive/negative moments. Be sure to record in Fig. 5 the end moment values and any other relevant moment values along the length of all elements after supplying calculations for their determination.
2. Determine the value for the translation at nodes 2 and 4.
3. Draw as accurately as possible in qualitative fashion the deformed shape of the *prestressed* structure under w in Fig. 6.

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1. Problem (50% weight)

The braced frame model in Fig. 1 is subjected to a horizontal force of 50 units and a vertical force of 60 units. The frame elements a through d have flexural capacity $M_p = 150$ units and very large axial capacity. Moreover, the effect of the axial force on the flexural capacity can be neglected. The truss elements e and f have the following axial capacities: $N_{pe} = 30$ units and $N_{pf} = 100$ units.

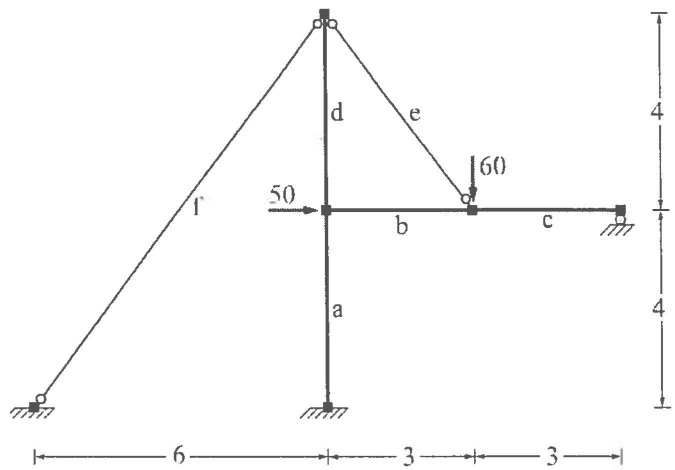

Figure 1: Braced frame model

Establish the critical collapse mechanism of the braced frame considering that the brace element f does not reach its capacity and determine the collapse load factor $\lambda_c \geq 0$ for the given reference loading consisting of the horizontal force of 50 units and the vertical force of 60 units.

2. Problem (50% weight)

The braced frame in Fig. 2 consists of the *inextensible* frame elements a and b and the truss element c. The flexural stiffness of the frame element a is EI_a and that of the frame element b is EI_b with $\eta = EI_b/EI_a = \frac{2}{3}$. The truss element has axial stiffness EA.

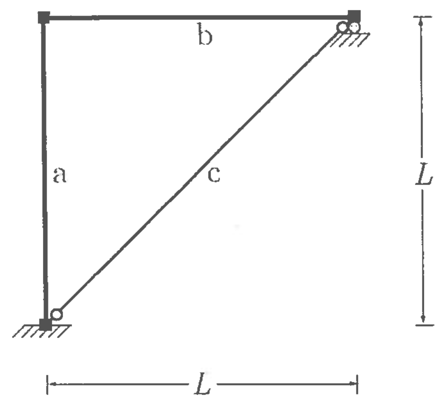

Figure 2: Frame with prestressed brace

Following the prestressing of the truss element c with an initial axial force q_o, it is connected to the frame made up of the frame elements a and b and released. In the final state the axial force in the truss element c measures $0.8093\, q_o$.

You are asked to determine the stiffness ratio γ between the axial stiffness EA of the truss element c and the flexural stiffness EI_a where

$$\gamma = \frac{EA}{EI_a/L^2}$$