Name: ________________________________

University of California, Berkeley
Civil and Environmental Engineering

Spring Semester, 2020

PhD Preliminary Examination – Dynamics

Do all three problems.

Problem 1 (25% weight)

Figure 1b shows the free vibration response (in inches) of the single degree of freedom system in Figure 1a. The mass *m* is 100 lb/g. The mass is at rest at time $t = 0$.

(a) Determine the natural frequency, ω_n, and the fraction of critical damping, ζ.
(b) Determine the energy dissipated by viscous damping from $t = 0$ to $t = 50$ seconds.
(c) If the mass were excited with a harmonic force with a maximum magnitude 5 lbs, sketch a plot of the maximum deformation as a function of the forcing frequency ω and label 3 points on the sketch.

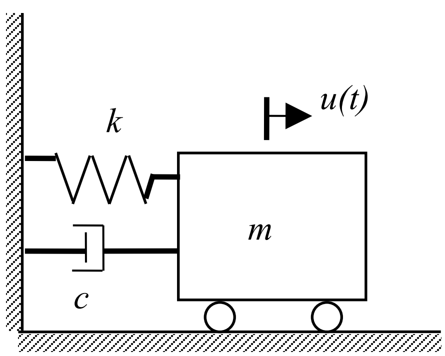

Figure 1a

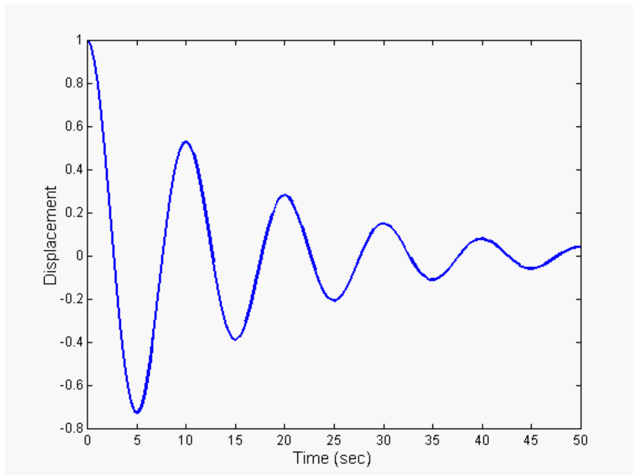

Figure 1b

Name: ______________________________

University of California, Berkeley
Civil and Environmental Engineering

Spring Semester, 2020

Problem 2 (35% weight)

A simply supported bridge with a single span of L feet has a deck of uniform cross section with mass m per foot length and flexural rigidity EI. A single wheel load p_o travels across the bridge at a uniform velocity of v, as shown in Figure 2.

The properties of the bridge are:
L = 100 ft, m = 11 kips/g per foot, I = 350 ft^4, and E = 576,000 kips/ft^2.

Assume the maximum speed of the wheel is v = 200 mph.

(a) Considering only the first mode response, and assuming that the shape function is sinusoidal, estimate the amplitude and location of the maximum dynamic displacement of the bridge.
(b) Considering only the second mode response, and assuming that the shape function is sinusoidal, estimate the amplitude and location of the maximum dynamic displacement of the bridge.

**Hints:
See Example 8.4 in the textbook.
Figures 2b and 2c, below, may be useful

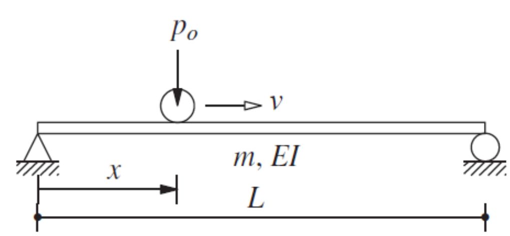

Figure 2a

Name: ______________________________

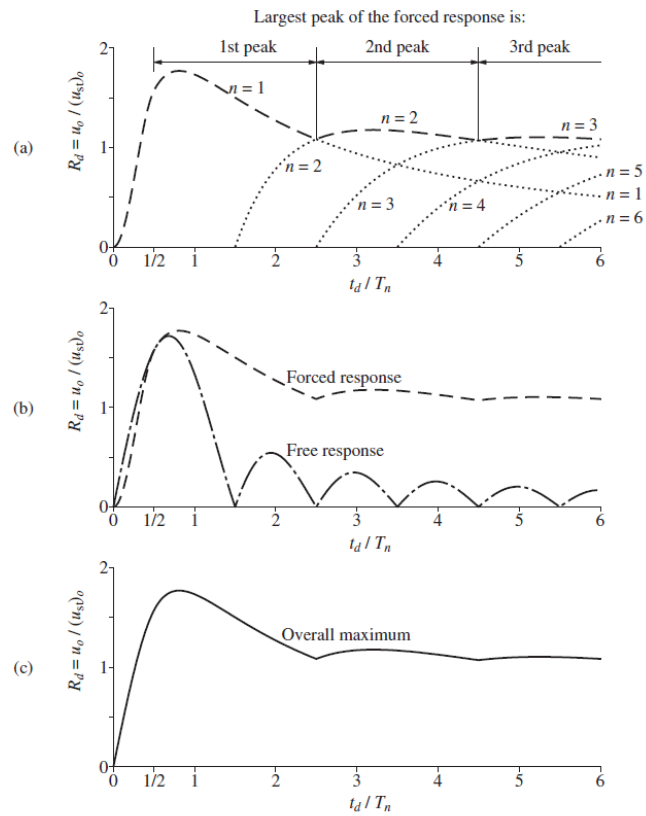

Figure 2b: Response to half cycle sine pulse

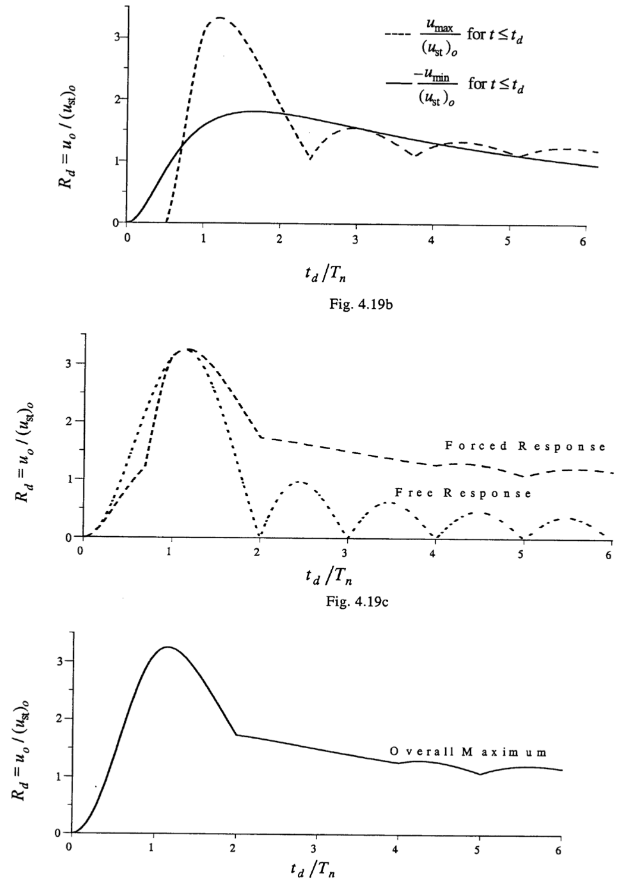

Figure 2c: Response to full cycle sine pulse

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University of California, Berkeley
Civil and Environmental Engineering

Spring Semester, 2020

Problem 3 (40% weight)

A two-story sway frame is shown in Figure 3a. The weight of each storey is 4500 lb and all columns have a flexural stiffness of $EI = 250$ kip-ft^2 and an elastic shear capacity of 1000 lb. There is a pin-joint at point A; all other connections are fixed.

(a) Find the natural frequencies and mode shapes.
(b) The building has 5% damping and experiences an earthquake with the response spectra shown in Figure 3b. Determine the maximum column shear. Would the structure remain elastic?
(c) Now assume the structure in Figure 3a is being designed using the inelastic design spectra in Figure 3c. The expected earthquake has a peak ground acceleration of 0.5g. Considering only the first mode, determine the required ductility factor μ.

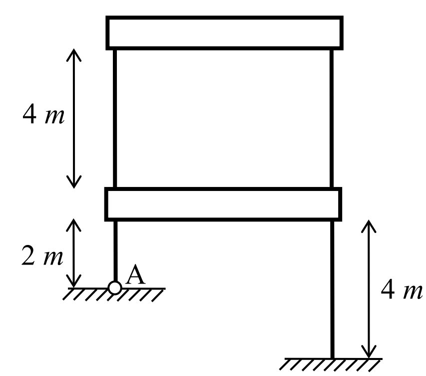

Figure 3a

Name: ________________________________

Damping, ζ = 0, 2, 5, 10, 20%

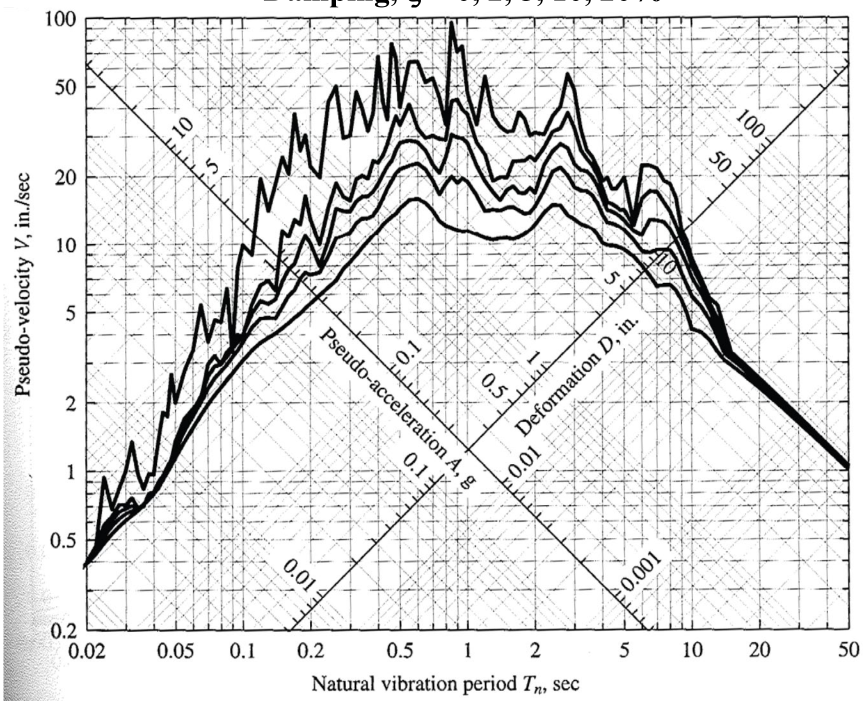

Figure 3b

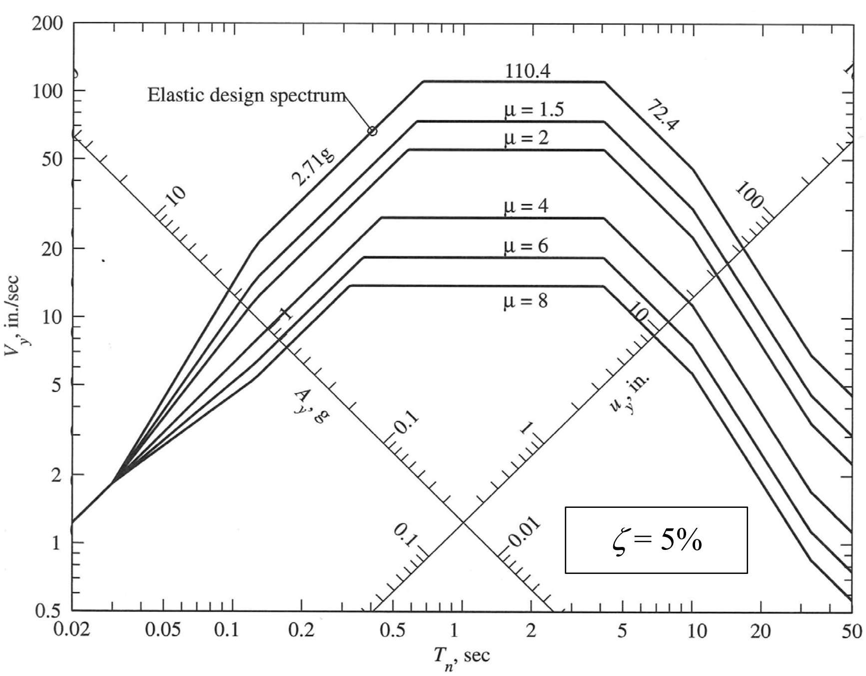

Figure 3c

Name: ______________________________

University of California, Berkeley
Civil and Environmental Engineering

Fall Semester, 2019

PhD Preliminary Examination - Dynamics

Problem 1 (30% weight)

Consider the steady-state motion of the single degree of freedom system shown in Figure 1, subjected to a forcing function of $p(t) = p_o \sin(\omega t)$.

p_o = 1 kips
$\omega / \omega_n = 2$
m = 2 kips/g
k_1 = 1000 kips/ft
k_2 = 3000 kips/ft
ζ = 2%

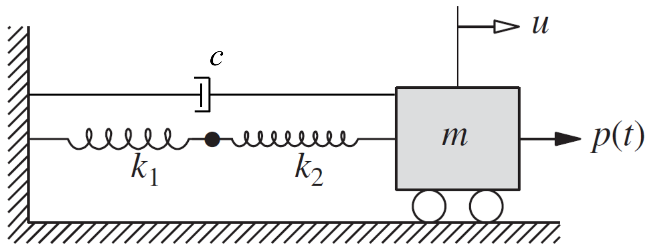

Figure 1

(a) Derive the energy dissipated by viscous damping in one cycle of vibration.
(b) Determine the maximum force transmitted to the support.

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University of California, Berkeley
Civil and Environmental Engineering

Fall Semester, 2019

Problem 2 (30% weight)

The two identical, rigid rods in Figure 2 are connected by a pin (i.e. hinge) and are each supported by a vertical spring at their center. Each rod has a mass m and rotational inertia (about the centroid of the rod) of J in the plane of the paper. The system has 3 degrees of freedom as labelled in Figure 3 as u_1, θ_2 and θ_3 . The subscripts indicate the number of the degree of freedom.

Assume there is a horizontal constraint at the pin so the system cannot move horizontally.
Assume small rotations.

Derive the first column of the mass matrix and the first column of the stiffness matrix.

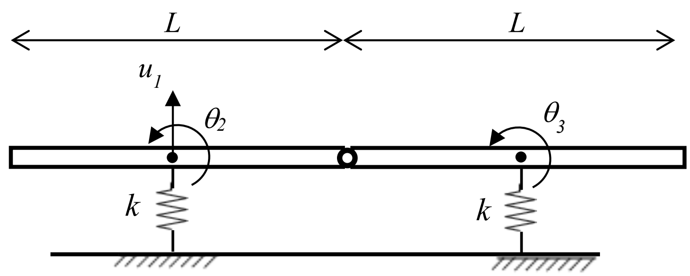


Figure 2

Name: ______________________________

University of California, Berkeley
Civil and Environmental Engineering

Fall Semester, 2019

<u>Problem 3</u> (40% weight)

The frame in Figure 3a has two elements, each with a flexural stiffness of *EI*. The frame is massless with lumped masses at the two nodes as shown. Axial deformations are to be neglected. For the two degrees of freedom shown in the figure (u_1 and u_2), the mass and stiffness matrices are given in Figure 3a.

m = 5 kips/g
$EI = 7\text{x}10^3$ k-ft^2
L = 10 ft

Considering <u>only</u> the first mode of response, determine the design overturning moment at the base for a vertical PGA of 0.2g.

Assume the first mode has 5% damping and the design spectrum in Figure 3b applies.

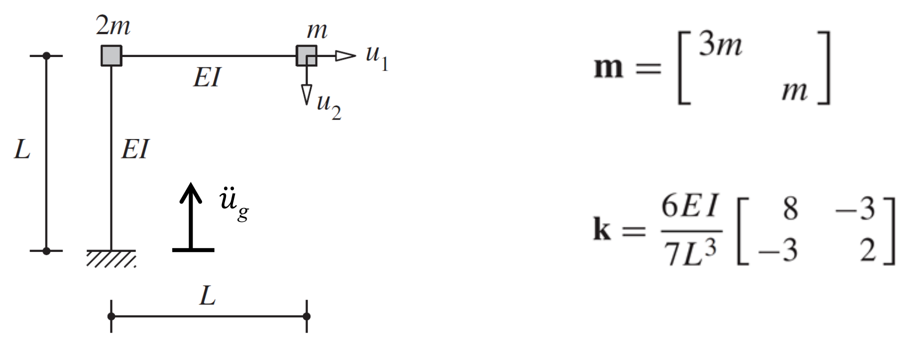


Figure 3a

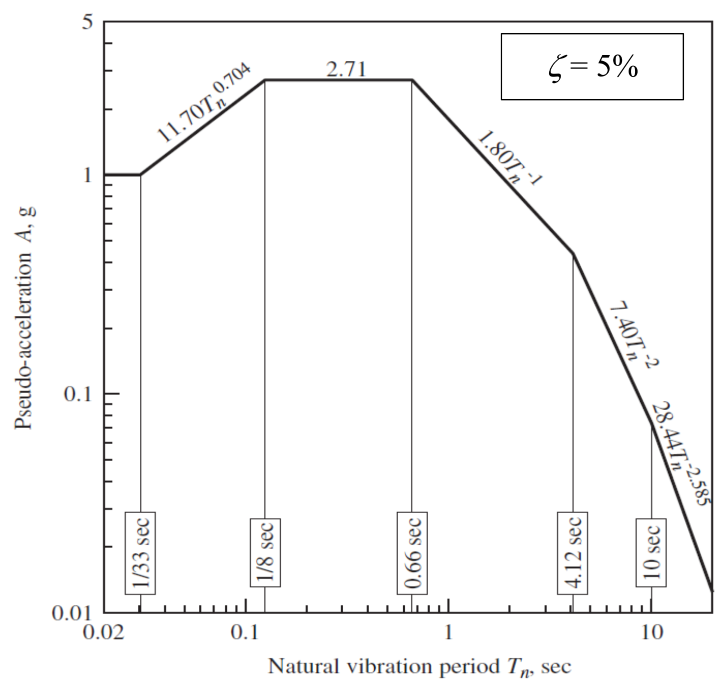


Figure 3b

Basic Definitions and Equations

$$\omega_n = \sqrt{\frac{k}{m}} \qquad T_n = \frac{2\pi}{\omega_n} \qquad f_n = \frac{1}{T_n} \qquad (u_{st})_o = \frac{p_o}{k}$$

$$\varsigma = \frac{c}{2m\omega_n} = \frac{c}{c_{cr}} \qquad \begin{array}{ll} \varsigma > 1 & \text{Overdamped} \\ \varsigma = 1 & \text{Critically damped} \\ \varsigma < 1 & \text{Underdamped} \end{array}$$

$$\omega_D = \omega_n\sqrt{1-\varsigma^2}$$

Free Vibration $m\ddot{u} + c\dot{u} + ku = 0$

For $\varsigma = 0$: $\quad u(t) = u(0)\cos\omega_n t + \dfrac{\dot{u}(0)}{\omega_n}\sin\omega_n t$

For $0 < \varsigma < 1$:

$$u(t) = e^{-\varsigma\omega_n t}\left(u(0)\cos\omega_D t + \frac{\dot{u}(0) + \varsigma\omega_n u(0)}{\omega_D}\sin\omega_D t\right)$$

Decay of Motion

$$\frac{u_1}{u_{j+1}} = \exp\left(\frac{2j\pi\varsigma}{\sqrt{1-\varsigma^2}}\right)$$

Free Vibration Test

$$\varsigma = \frac{1}{2\pi j}\ln\frac{u_i}{u_{i+j}}$$

Harmonic Excitation $m\ddot{u} + c\dot{u} + ku = p_o \sin\omega t$

Steady State Response

$$u(t) = u_o \sin(\omega t - \phi)$$

$$R_d = \frac{u_o}{(u_{st})_o} = \frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + [2\zeta(\omega/\omega_n)]^2}}$$

$$\varphi = \tan^{-1}\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}$$

Resonance at $\omega_n\sqrt{1-2\zeta^2}$ with $R_d = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

Half-Power Bandwidth

$$\frac{\omega_b - \omega_a}{\omega_n} = 2\zeta$$

Vibration Generator: $p(t) = (m_e e\omega^2)\sin\omega t$

Transmissibility $TR = (f_T)_o / p_o = \ddot{u}_o^t / \ddot{u}_{go}$

$$TR = R_d\sqrt{1 + [2\zeta(\omega/\omega_n)]^2}$$

Equivalent Viscous Damping

$$\zeta_{eq} = \frac{1}{4\pi}\frac{E_D}{E_{So}}$$

Arbitrary Excitation $m\ddot{u} + c\dot{u} + ku = p(t)$

Reponse to unit Impulse: $p(t) = \delta(t-\tau)$

$$h(t-\tau) \equiv u(t) = \frac{1}{m\omega_n}\sin(\omega_n(t-\tau)) \qquad \zeta = 0$$

Duhamel's Integral

$$u(t) = \int_0^t p(\tau)h(t-\tau)d\tau$$

Response to Step Force, $\varsigma = 0$

$$u(t) = (u_{st})_o(1 - \cos\omega_n t)$$

Response to Ramp Force, $\varsigma = 0$

$$p(t) = p_o\frac{t}{t_r} \qquad u(t) = (u_{st})_o\left(\frac{t}{t_r} - \frac{\sin\omega_n t}{\omega_n t_r}\right)$$

Response to Rectangular Pulse

$$R_d = \begin{cases} 2\sin\pi t_d/T_n & t_d/T_n \le \frac{1}{2} \\ 2 & t_d/T_n \ge \frac{1}{2} \end{cases}$$

Short Pulse $\quad I = \int p(t)dt \qquad u(t) = I\left(\dfrac{1}{m\omega_n}\sin\omega_n t\right)$

Earthquake Response $p(t) = p_{eff}(t) = -m\ddot{u}_g(t)$

$$u \equiv u(t, T_n, \zeta) \qquad u_o \equiv u_o(T_n, \zeta)$$

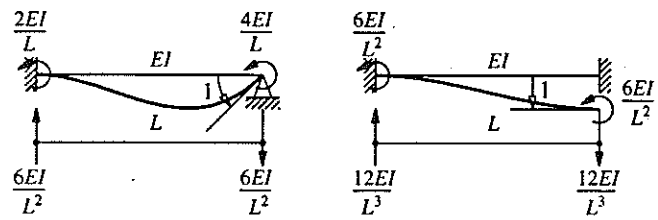
$$D \equiv u_o \qquad V = \omega_n D \qquad A = \omega_n^2 D$$

$$\frac{A}{\omega_n} = V = \omega_n D \qquad E_{so} = \frac{mV^2}{2} \qquad f_{so} = kD = mA$$

For One Story Structure

$$V_{bo} = f_{so} = \frac{A}{g}w \qquad M_{bo} = hV_{bo}$$

Stiffness Coefficients for a Flexural Element



EQ Reponse of Inelastic Systems $m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = p_{eff}(t) = -m\ddot{u}_g(t)$

Normalized Yield Strength

$$\bar{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o}$$

f_y and u_y are yield strength and yield deformation

f_o and u_o are peak force and deformation in corresponding linear system

Yield Strength Reduction Factor

$$R_y = \frac{f_o}{f_y} = \frac{u_o}{u_y} \qquad f_o = ku_o$$

f_o is minimum strength required for structure to remain elastic

Ductility Factor

$$\mu = \frac{u_m}{u_y}$$

u_m is peak deformation of elastoplastic system

Response Spectrum for Inelastic Systems

$$D_y = u_y \qquad V_y = \omega_n D_y \qquad A_y = \omega_n^2 D_y$$

$$f_y = \frac{A_y}{g}w \qquad u_m = \mu\left(\frac{T_n}{2\pi}\right)^2 A_y$$

Generalized SDOF Systems: Distributed Mass and Elasticity

For Assumed Shape Function $\psi(x)$

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx \qquad \tilde{\Gamma} = \frac{\tilde{L}}{\tilde{m}} \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{L} = \int_0^L m(x)\psi(x)dx \qquad z_o = \tilde{\Gamma}D$$

At Height x Above the Base

$$u_o(x) = \tilde{\Gamma}D\psi(x) \qquad f_o(x) = \tilde{\Gamma}m(x)\psi(x)A$$

Static Analysis of the tower due to $f_o(x)$ provides interal forces.

Base Shear and Moment:

$$V_{bo} = V_o(0) = \tilde{L}\tilde{\Gamma}A \qquad M_{bo} = M_o(0) = \tilde{L}^{\theta}\tilde{\Gamma}A$$

$$\tilde{L}^{\theta} = \int_0^L xm(x)\psi(x)dx$$

Name: ______________________________

University of California, Berkeley
Civil and Environmental Engineering

Fall Semester, 2018

Preliminary Examination - Dynamics

Problem 1 (30% weight)

An undamped SDOF system with mass *m* and stiffness *k* is initially at rest and is then subjected to a full-cycle sine pulse of ground motion, as shown in Figure 1. The natural period of the system is $T_n = t_d/2$. Determine the deformation, $u(t)$, for the time interval $0 < t < t_d$.

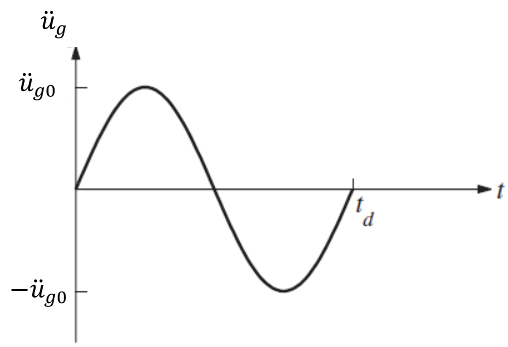


Figure 1

Problem 2 (30% weight)

The beam in Figure 2 has a uniform mass per unit length of $m(x) = m$ and a uniform bending stiffness of $EI(x) = EI$. Using Rayleigh's method, estimate the fundamental natural frequency of the beam.

Recall: $$\tilde{m} = \int_0^L m(x)[\psi(x)]^2\,dx$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2\,dx$$

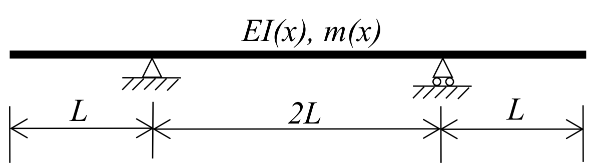


Figure 2

Name: ________________________________

University of California, Berkeley
Civil and Environmental Engineering

Fall Semester, 2018

Problem 3 (40% weight)

Figure 3a shows a small mass m and a large mass $5m$ that can only translate in the vertical direction.

(a) Determine the natural frequencies and mode shapes.

(b) The system experiences a vertical ground motion which can be described by the response spectra in Figure 3b. Using modal superposition, estimate the maximum force in the spring between the two masses during the earthquake. Assume that all modes of the system have 5% damping.

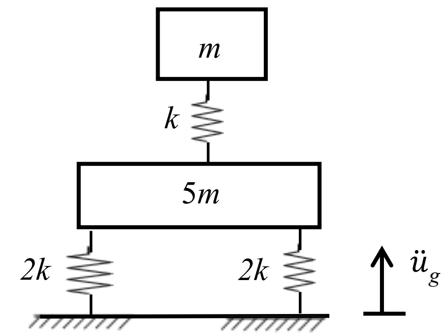
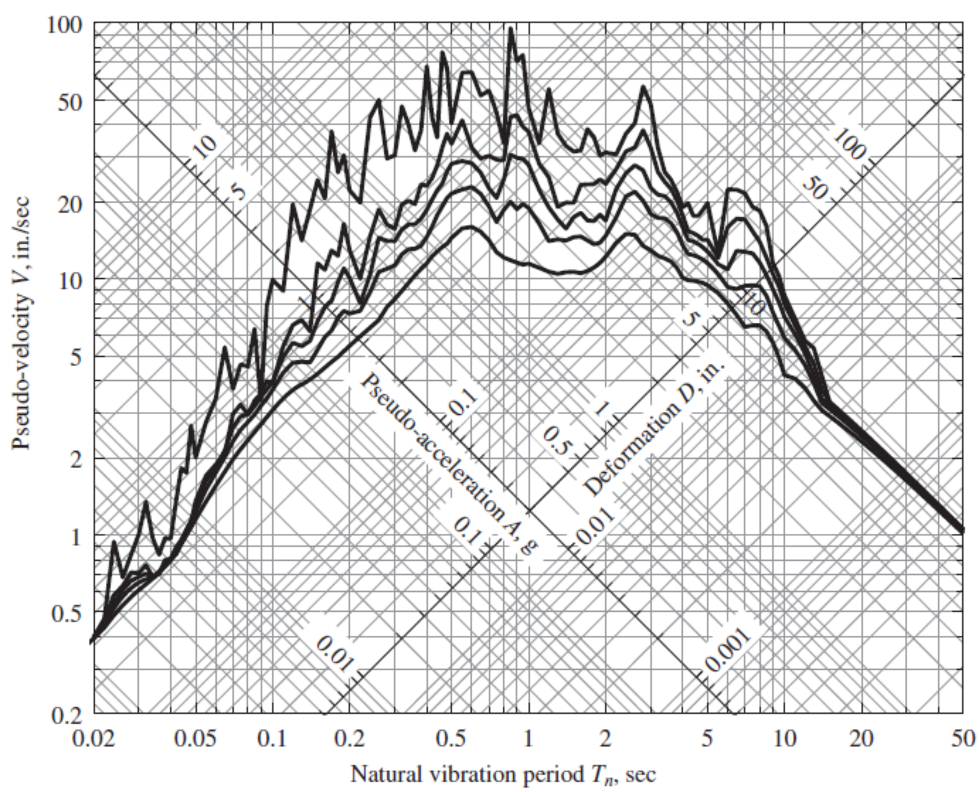


Figure 3a



* Spectra shown for ζ = 0, 2, 5, 10, 20%

Figure 3b

UNIVERSITY OF CALIFORNIA, BERKELEY **Dept. of Civil and Environmental Engineering**
Spring Semester 2018 **Structural Engineering, Mechanics and Materials**

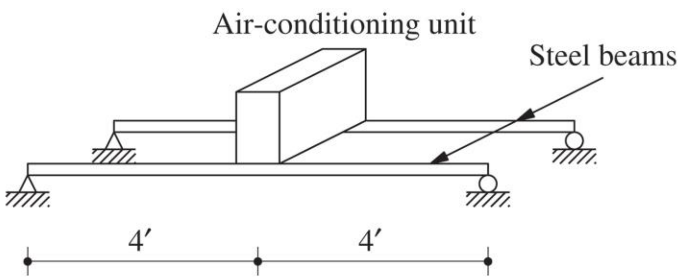
Ph.D. Preliminary Examination: Dynamics

Name: ________________________________

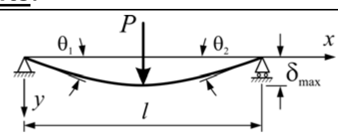
QUESTION 1 [30%]:

An air-conditioning unit weighing 1200 lb is bolted at the middle of two parallel simply supported steel beams (E = 30,000 ksi) as shown below. The clear span of the beams is L = 8 ft. The second moment of cross-sectional area of each beam is I = 10 in^4. The motor in the unit runs at 300 rpm and produces an unbalanced vertical force of 60 lb at this speed. Neglect the weight of the beams and assume ζ = 1% viscous damping in the system. Determine the amplitudes of:

1) steady-state deflection, and 2) steady-state acceleration (in g's) of the beams at their midpoints which result from the unbalanced force.

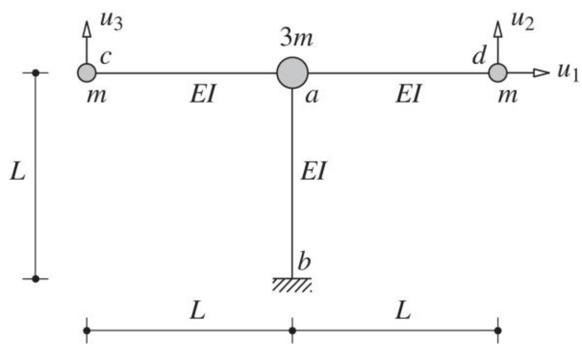


Hints:

 $\delta_{max} = \dfrac{Pl^3}{48EI}$	Deformation response factor: $R_d = \dfrac{1}{\sqrt{\left[1 - (\omega/\omega_n)^2\right]^2 + \left[2\zeta\omega/\omega_n\right]^2}}$	g = 386 in/sec^2

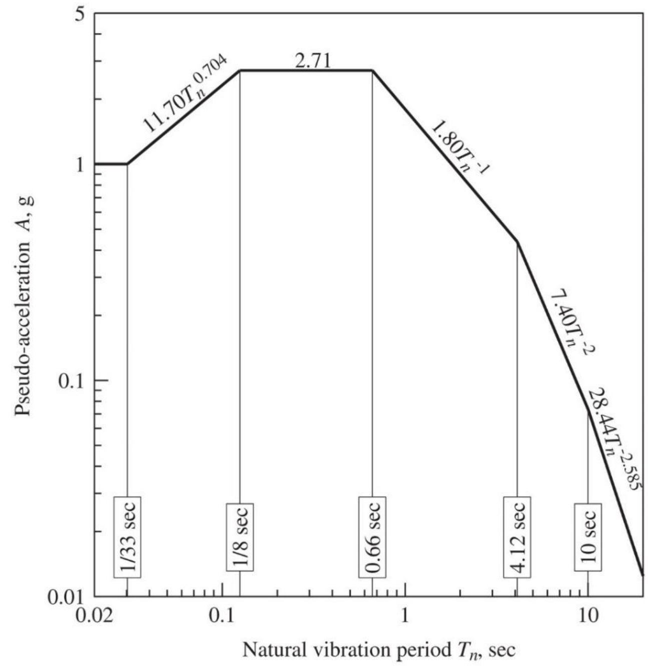
<u>QUESTION 2 [70%]</u>:

The umbrella structure shown below has 3 DOF with the listed eigen solution. It is made of standard steel pipe and has the following properties: I = 28.1 in^4, E = 29,000 ksi, weight = 18.97 lb/ft, m = 1.5 kips/g, and L = 10 ft. It is required to determine the peak response of this structure to horizontal ground motion characterized by the design spectrum shown below (for 5% damping) scaled to 0.20g peak ground acceleration and using the SRSS modal combination rule. It is required to perform the following: 1) Check that the distributed weight of the umbrella structure can be neglected (i.e. less than 10%) compared to the lumped weights and find the corresponding mass matrix, 2) Compute the displacements u_1, u_2, and u_3, and 3) Computed the bending moments at the base of the column "b" and at location "a" of the beam.



$$\omega_1 = 0.5259\sqrt{\frac{EI}{mL^3}} \quad \omega_2 = 1.6135\sqrt{\frac{EI}{mL^3}} \quad \omega_3 = 1.7321\sqrt{\frac{EI}{mL^3}}$$

$$\phi_1 = \begin{Bmatrix} 1 \\ -1.9492 \\ 1.9492 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} 1 \\ 1.2826 \\ -1.2826 \end{Bmatrix} \quad \phi_3 = \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$



Elastic pseudo-acceleration design spectrum for ground motion: $\ddot{u}_{go}$ = 1g (g = 386 in/sec^2) and ζ = 5%.

Basic Definitions and Equations

$$\omega_n = \sqrt{\frac{k}{m}} \qquad T_n = \frac{2\pi}{\omega_n} \qquad f_n = \frac{1}{T_n} \qquad (u_{st})_o = \frac{p_o}{k}$$

$$\varsigma = \frac{c}{2m\omega_n} = \frac{c}{c_{cr}}$$

$\varsigma > 1$ Overdamped

$\varsigma = 1$ Critically damped

$\varsigma < 1$ Underdamped

$$\omega_D = \omega_n\sqrt{1-\varsigma^2}$$

Free Vibration $m\ddot{u} + c\dot{u} + ku = 0$

For $\varsigma = 0$: $\quad u(t) = u(0)\cos\omega_n t + \dfrac{\dot{u}(0)}{\omega_n}\sin\omega_n t$

For $0 < \varsigma < 1$:

$$u(t) = e^{-\varsigma\omega_n t}\left(u(0)\cos\omega_D t + \frac{\dot{u}(0) + \varsigma\omega_n u(0)}{\omega_D}\sin\omega_D t \right)$$

Decay of Motion

$$\frac{u_1}{u_{j+1}} = \exp\left(\frac{2j\pi\varsigma}{\sqrt{1-\varsigma^2}}\right)$$

Free Vibration Test

$$\varsigma = \frac{1}{2\pi j}\ln\frac{u_i}{u_{i+j}}$$

Harmonic Excitation $m\ddot{u} + c\dot{u} + ku = p_o \sin\omega t$

Steady State Response

$$u(t) = u_o \sin(\omega t - \phi)$$

$$R_d = \frac{u_o}{(u_{st})_o} = \frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + [2\zeta(\omega/\omega_n)]^2}}$$

$$\varphi = \tan^{-1}\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}$$

Resonance at $\omega_n\sqrt{1-2\zeta^2}$ with $R_d = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

Half-Power Bandwidth

$$\frac{\omega_b - \omega_a}{\omega_n} = 2\zeta$$

Vibration Generator: $\quad p(t) = (m_e e\omega^2)\sin\omega t$

Transmissibility $\quad TR = (f_T)_o / p_o = \ddot{u}_o^t / \ddot{u}_{go}$

$$TR = R_d\sqrt{1 + [2\zeta(\omega/\omega_n)]^2}$$

Equivalent Viscous Damping

$$\zeta_{eq} = \frac{1}{4\pi}\frac{E_D}{E_{So}}$$

Arbitrary Excitation $m\ddot{u} + c\dot{u} + ku = p(t)$

Reponse to unit Impulse: $\quad p(t) = \delta(t-\tau)$

$$h(t-\tau) \equiv u(t) = \frac{1}{m\omega_n}\sin(\omega_n(t-\tau)) \qquad \zeta = 0$$

Duhamel's Integral

$$u(t) = \int_0^t p(\tau)h(t-\tau)d\tau$$

Response to Step Force, $\varsigma = 0$

$$u(t) = (u_{st})_o(1-\cos\omega_n t)$$

Response to Ramp Force, $\varsigma = 0$

$$p(t) = p_o\frac{t}{t_r} \qquad u(t) = (u_{st})_o\left(\frac{t}{t_r} - \frac{\sin\omega_n t}{\omega_n t_r}\right)$$

Response to Rectangular Pulse

$$R_d = \begin{cases} 2\sin\pi t_d/T_n & t_d/T_n \le \frac{1}{2} \\ 2 & t_d/T_n \ge \frac{1}{2} \end{cases}$$

Short Pulse $\quad I = \int p(t)dt \qquad u(t) = I\left(\dfrac{1}{m\omega_n}\sin\omega_n t\right)$

Earthquake Response $p(t) = p_{eff}(t) = -m\ddot{u}_g(t)$

$$u \equiv u(t, T_n, \zeta) \qquad u_o \equiv u_o(T_n, \zeta)$$

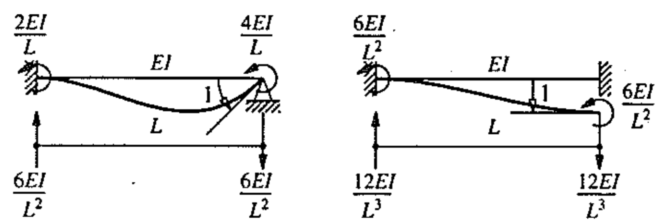
$$D \equiv u_o \qquad V = \omega_n D \qquad A = \omega_n^2 D$$

$$\frac{A}{\omega_n} = V = \omega_n D \qquad E_{so} = \frac{mV^2}{2} \qquad f_{so} = kD = mA$$

For One Story Structure

$$V_{bo} = f_{so} = \frac{A}{g}w \qquad M_{bo} = hV_{bo}$$

Stiffness Coefficients for a Flexural Element



EQ Reponse of Inelastic Systems $m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = p_{eff}(t) = -m\ddot{u}_g(t)$

Normalized Yield Strength

$$\bar{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o}$$

f_y and u_y are yield strength and yield deformation

f_o and u_o are peak force and deformation in corresponding linear system

Yield Strength Reduction Factor

$$R_y = \frac{f_o}{f_y} = \frac{u_o}{u_y} \qquad f_o = ku_o$$

f_o is minimum strength required for structure to remain elastic

Ductility Factor

$$\mu = \frac{u_m}{u_y}$$

u_m is peak deformation of elastoplastic system

Response Spectrum for Inelastic Systems

$$D_y = u_y \qquad V_y = \omega_n D_y \qquad A_y = \omega_n^2 D_y$$

$$f_y = \frac{A_y}{g}w \qquad u_m = \mu\left(\frac{T_n}{2\pi}\right)^2 A_y$$

Generalized SDOF Systems: Distributed Mass and Elasticity

For Assumed Shape Function $\psi(x)$

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx \qquad \tilde{\Gamma} = \frac{\tilde{L}}{\tilde{m}} \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{L} = \int_0^L m(x)\psi(x)dx \qquad z_o = \tilde{\Gamma}D$$

At Height x Above the Base

$$u_o(x) = \tilde{\Gamma}D\psi(x) \qquad f_o(x) = \tilde{\Gamma}m(x)\psi(x)A$$

Static Analysis of the tower due to $f_o(x)$ provides interal forces.

Base Shear and Moment:

$$V_{bo} = V_o(0) = \tilde{L}\tilde{\Gamma}A \qquad M_{bo} = M_o(0) = \tilde{L}^\theta\tilde{\Gamma}A$$

$$\tilde{L}^\theta = \int_0^L x m(x)\psi(x)dx$$