Name: ______________________________

University of California, Berkeley
Civil and Environmental Engineering

Fall Semester, 2021

PhD Preliminary Examination - Dynamics

Problem 1 (20% weight)

A single degree of freedom structure has a height of 330 feet, a bending stiffness of $EI = 2\text{x}10^{11}$ kip-ft^2, and a weight of W = 9,000 kips, as shown in Figure 1a. Assume that the structure is clamped at the base and the rotational inertia of the mass about its center is zero.

(a) Determine the maximum shear in the structure due to the impulse of horizontal ground acceleration shown in Figure 1(b). You may use the shock spectrum in Figure 4.9.3 below which is applicable for zero damping.

(b) Now assume the structure has 5% damping. Explain how you would modify your result from part (a). You do not need to do any calculations.

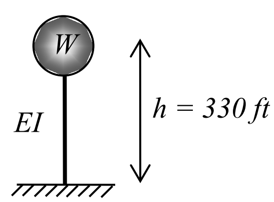

Figure 1a

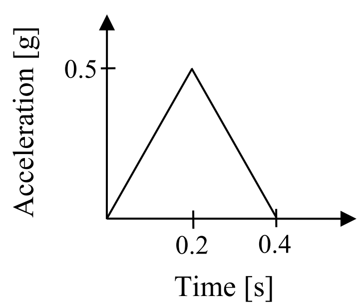

Figure 1b

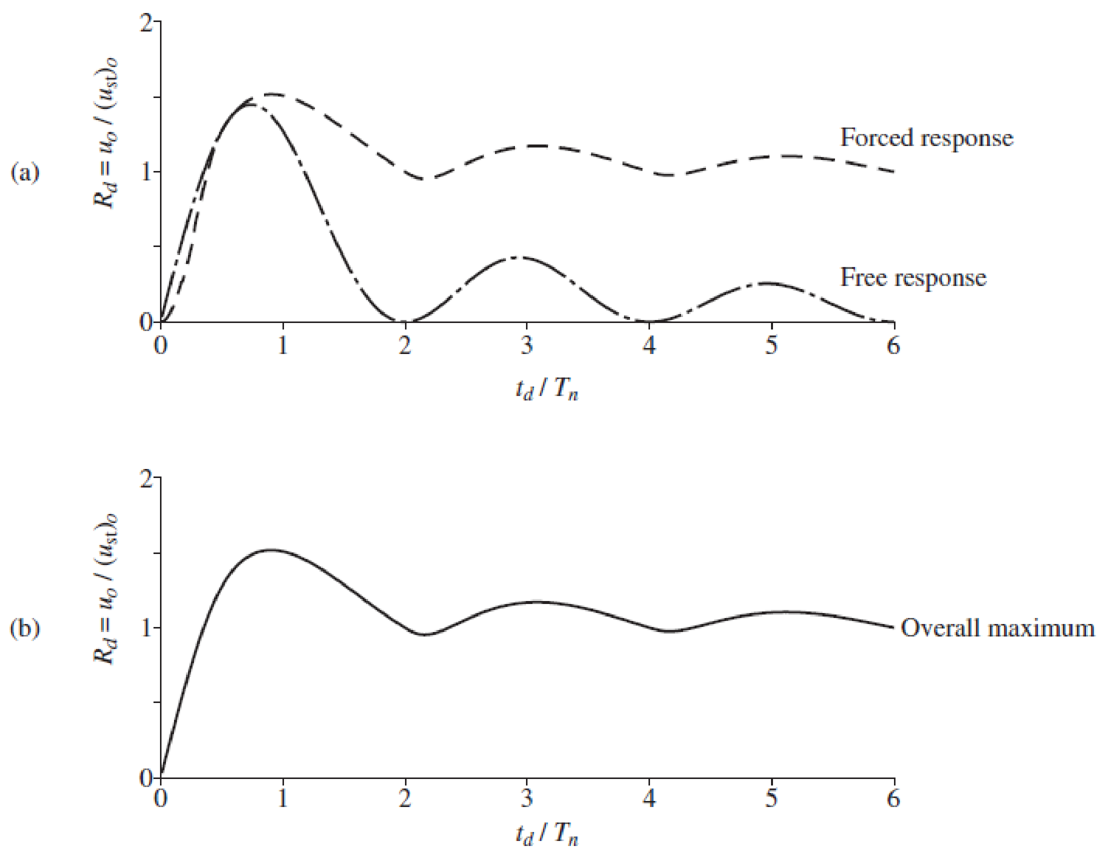

Figure 4.9.3 Response to triangular pulse force: (a) maximum response during each of forced vibration and free vibration phases; (b) shock spectrum.

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Problem 2 (45% weight)

(a) You are asked to design a base isolation system for the SDOF structure from Problem 1, as shown in Figure 2. The weight of the base is 2/3 of the weight of the superstructure ($W_b = 2W/3$). Determine the spring stiffness, k_b, required to increase the fundamental natural period of the structure to 4 times the natural period of the structure from Problem 1.

(b) The design spectrum is shown in Figure 4b for a damping ratio of $\zeta = 5\%$. The PGA for the site is 0.6g. Assuming that the damper can be designed to provide 5% damping in the first mode, determine the maximum shear force in the structure due to the first mode response only.

(c) Explain the additional steps you would take to include both modes when determining the maximum shear force in the structure. You do not need to do any calculations but please use equations to explain the steps.

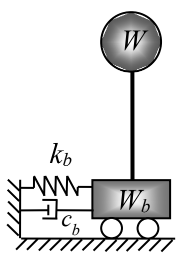

Figure 2a

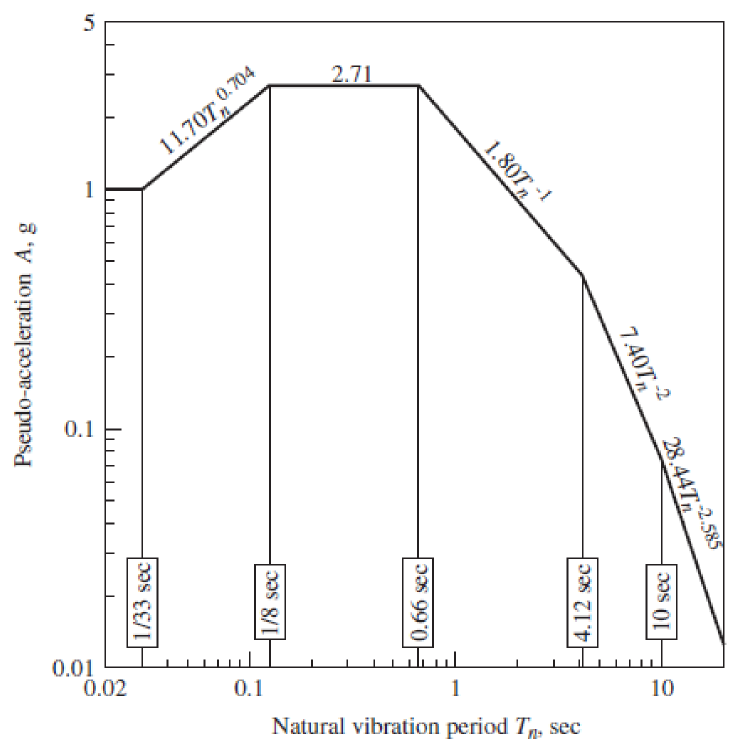

Figure 2b

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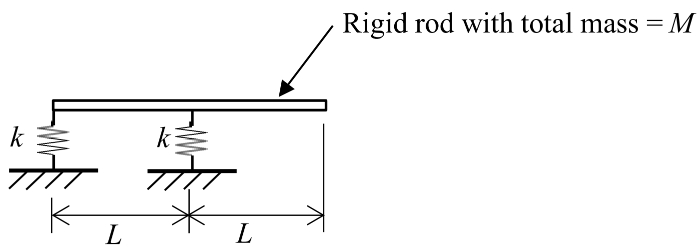
Problem 3 (35% weight)

The rigid rod shown in Figure 3 has a total mass of M and is supported by two springs of stiffness k. The center of the rod is prevented from translating horizontally.

Derive the equations of motion for free vibration. Clearly define the coordinates (i.e. the locations of the displacements or rotations) that you use in your derivation, including where they are zero. Assume small rotations.



Basic Definitions and Equations

$$\omega_n = \sqrt{\frac{k}{m}} \qquad T_n = \frac{2\pi}{\omega_n} \qquad f_n = \frac{1}{T_n} \qquad (u_{st})_o = \frac{p_o}{k}$$

$$\varsigma = \frac{c}{2m\omega_n} = \frac{c}{c_{cr}}$$

$\varsigma > 1$ Overdamped

$\varsigma = 1$ Critically damped

$\varsigma < 1$ Underdamped

$$\omega_D = \omega_n\sqrt{1-\varsigma^2}$$

Free Vibration $m\ddot{u} + c\dot{u} + ku = 0$

For $\varsigma = 0$: $\quad u(t) = u(0)\cos\omega_n t + \dfrac{\dot{u}(0)}{\omega_n}\sin\omega_n t$

For $0 < \varsigma < 1$:

$$u(t) = e^{-\varsigma\omega_n t}\left(u(0)\cos\omega_D t + \frac{\dot{u}(0) + \varsigma\omega_n u(0)}{\omega_D}\sin\omega_D t\right)$$

Decay of Motion

$$\frac{u_1}{u_{j+1}} = \exp\left(\frac{2j\pi\varsigma}{\sqrt{1-\varsigma^2}}\right)$$

Free Vibration Test

$$\varsigma = \frac{1}{2\pi j}\ln\frac{u_i}{u_{i+j}}$$

Harmonic Excitation $m\ddot{u} + c\dot{u} + ku = p_o\sin\omega t$

Steady State Response

$$u(t) = u_o\sin(\omega t - \phi)$$

$$R_d = \frac{u_o}{(u_{st})_o} = \frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + [2\zeta(\omega/\omega_n)]^2}}$$

$$\varphi = \tan^{-1}\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}$$

Resonance at $\omega_n\sqrt{1-2\zeta^2}$ with $R_d = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

Half-Power Bandwidth

$$\frac{\omega_b - \omega_a}{\omega_n} = 2\zeta$$

Vibration Generator: $p(t) = (m_e e\omega^2)\sin\omega t$

Transmissibility $TR = (f_T)_o / p_o = \ddot{u}_o^t / \ddot{u}_{go}$

$$TR = R_d\sqrt{1 + [2\zeta(\omega/\omega_n)]^2}$$

Equivalent Viscous Damping

$$\zeta_{eq} = \frac{1}{4\pi}\frac{E_D}{E_{So}}$$

Arbitrary Excitation $m\ddot{u} + c\dot{u} + ku = p(t)$

Reponse to unit Impulse: $p(t) = \delta(t-\tau)$

$$h(t-\tau) \equiv u(t) = \frac{1}{m\omega_n}\sin(\omega_n(t-\tau)) \qquad \zeta = 0$$

Duhamel's Integral

$$u(t) = \int_0^t p(\tau)h(t-\tau)d\tau$$

Response to Step Force, $\varsigma = 0$

$$u(t) = (u_{st})_o(1-\cos\omega_n t)$$

Response to Ramp Force, $\varsigma = 0$

$$p(t) = p_o\frac{t}{t_r} \qquad u(t) = (u_{st})_o\left(\frac{t}{t_r} - \frac{\sin\omega_n t}{\omega_n t_r}\right)$$

Response to Rectangular Pulse

$$R_d = \begin{cases} 2\sin\pi t_d/T_n & t_d/T_n \le \frac{1}{2} \\ 2 & t_d/T_n \ge \frac{1}{2} \end{cases}$$

Short Pulse $\quad I = \int p(t)dt \qquad u(t) = I\left(\dfrac{1}{m\omega_n}\sin\omega_n t\right)$

Earthquake Response $p(t) = p_{eff}(t) = -m\ddot{u}_g(t)$

$$u \equiv u(t, T_n, \zeta) \qquad u_o \equiv u_o(T_n, \zeta)$$

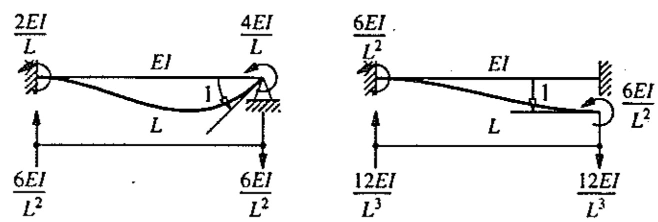
$$D \equiv u_o \qquad V = \omega_n D \qquad A = \omega_n^2 D$$

$$\frac{A}{\omega_n} = V = \omega_n D \qquad E_{so} = \frac{mV^2}{2} \qquad f_{so} = kD = mA$$

For One Story Structure

$$V_{bo} = f_{so} = \frac{A}{g}w \qquad M_{bo} = hV_{bo}$$

Stiffness Coefficients for a Flexural Element



EQ Reponse of Inelastic Systems $m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = p_{eff}(t) = -m\ddot{u}_g(t)$

Normalized Yield Strength

$$\bar{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o}$$

f_y and u_y are yield strength and yield deformation

f_o and u_o are peak force and deformation in corresponding linear system

Yield Strength Reduction Factor

$$R_y = \frac{f_o}{f_y} = \frac{u_o}{u_y} \qquad f_o = ku_o$$

f_o is minimum strength required for structure to remain elastic

Ductility Factor

$$\mu = \frac{u_m}{u_y}$$

u_m is peak deformation of elastoplastic system

Response Spectrum for Inelastic Systems

$$D_y = u_y \qquad V_y = \omega_n D_y \qquad A_y = \omega_n^2 D_y$$

$$f_y = \frac{A_y}{g}w \qquad u_m = \mu\left(\frac{T_n}{2\pi}\right)^2 A_y$$

Generalized SDOF Systems: Distributed Mass and Elasticity

For Assumed Shape Function $\psi(x)$

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx \qquad \tilde{\Gamma} = \frac{\tilde{L}}{\tilde{m}} \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{L} = \int_0^L m(x)\psi(x)dx \qquad z_o = \tilde{\Gamma}D$$

At Height x Above the Base

$$u_o(x) = \tilde{\Gamma}D\psi(x) \qquad f_o(x) = \tilde{\Gamma}m(x)\psi(x)A$$

Static Analysis of the tower due to $f_o(x)$ provides interal forces.

Base Shear and Moment:

$$V_{bo} = V_o(0) = \tilde{L}\tilde{\Gamma}A \qquad M_{bo} = M_o(0) = \tilde{L}^\theta\tilde{\Gamma}A$$

$$\tilde{L}^\theta = \int_0^L x m(x)\psi(x)dx$$

Equation of Motion, MDOF $\mathbf{m\ddot{u}+c\dot{u}+ku=p}(t)$

Earthquake Excitation

$$\mathbf{p}(t)=\mathbf{p}_{eff}(t)=-\mathbf{m}\,\iota\,\ddot{u}_g(t)$$

where $\iota \equiv$ influence vector

Static Condesation

$\mathbf{u}_t \equiv$ translational DOF $\mathbf{u}_o \equiv$ rotational DOF

$$\begin{bmatrix}\mathbf{m}_{tt} & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{u}}_t\\ \ddot{\mathbf{u}}_o\end{Bmatrix}+\begin{bmatrix}\mathbf{k}_{tt} & \mathbf{k}_{to}\\ \mathbf{k}_{ot} & \mathbf{k}_{oo}\end{bmatrix}\begin{Bmatrix}\mathbf{u}_t\\ \mathbf{u}_o\end{Bmatrix}=\begin{Bmatrix}\mathbf{p}_t(t)\\ \mathbf{0}\end{Bmatrix}$$

$$\hat{\mathbf{k}}_{tt}=\mathbf{k}_{tt}-\mathbf{k}_{ot}^T\mathbf{k}_{oo}^{-1}\mathbf{k}_{ot}$$

To find $\mathbf{u}_t(t)$ solve: $\mathbf{m}_{tt}\ddot{\mathbf{u}}_t+\hat{\mathbf{k}}_{tt}\mathbf{u}_t=\mathbf{p}_t(t)$

To find $\mathbf{u}_o(t)$ use: $\mathbf{u}_o=-\mathbf{k}_{oo}^{-1}\mathbf{k}_{ot}\mathbf{u}_t$

Natural Frequencies and Modes

To find ω_n^2, use characteristic equation: $\det\left[\mathbf{k}-\omega_n^2\mathbf{m}\right]=0$

To find ϕ_n, solve: $\left[\mathbf{k}-\omega_n^2\mathbf{m}\right]\phi_n=0$

Orthogonality of Modes

$$\phi_n^T\mathbf{k}\,\phi_r=0 \qquad \phi_n^T\mathbf{m}\,\phi_r=0$$

Modal Expansion of Displacements

$$\mathbf{u}=\sum_{r=1}^{N}\phi_r q_r=\Phi\mathbf{q} \qquad \text{where: } q_n=\frac{\phi_n^T\mathbf{m}\mathbf{u}}{M_n}$$

For Initial Conditions: $\mathbf{u}(0),\dot{\mathbf{u}}(0)$

$$q_n(0)=\frac{\phi_n^T\mathbf{m}\mathbf{u}(0)}{M_n} \qquad \dot{q}_n(0)=\frac{\phi_n^T\mathbf{m}\dot{\mathbf{u}}(0)}{M_n}$$

Modal Equations

$$M_n\ddot{q}_n+C_n\dot{q}_n+K_nq_n=P_n(t)$$

$$\ddot{q}_n+2\zeta_n\omega_n\dot{q}_n+\omega_n^2q_n=\frac{P_n(t)}{M_n}$$

$$K_n=\phi_n^T\mathbf{k}\,\phi_n \qquad M_n=\phi_n^T\mathbf{m}\,\phi_n \qquad P_n(t)=\phi_n^T\mathbf{p}(t)$$

For classical damping: $C_n=\phi_n^T\mathbf{c}\,\phi_n \qquad \varsigma_n=\dfrac{C_n}{2M_n\omega_n}$

Dynamic Response

Solve modal equations for $q_n(t)$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\mathbf{u}_n(t)=\sum_{n=1}^{N}\boldsymbol{\varphi}_n q_n(t)$$

Equivalent static forces, nth mode

$$\mathbf{f}_n(t)=\omega_n^2\mathbf{m}\boldsymbol{\varphi}_n q_n(t)$$

Internal forces by static analysis

EQ Response History Analysis $\mathbf{m\ddot{u}+c\dot{u}+ku}=\mathbf{p}_{eff}(t)=-\mathbf{m}\,\iota\,\ddot{u}_g(t)$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\phi_nq_n(t) \qquad \mathbf{m}\,\iota=\sum_{n=1}^{N}s_n=\sum_{n=1}^{N}\Gamma_n\mathbf{m}\,\phi_n$$

$$\Gamma_n=\frac{L_n}{M_n} \qquad L_n=\phi_n^T\mathbf{m}\,\iota \qquad M_n=\phi_n^T\mathbf{m}\,\phi_n$$

$$q_n(t)=\Gamma_nD_n(t)$$

Modal Equations for Classically-Damped Systems

$$\ddot{q}_n+2\varsigma_n\omega_n\dot{q}_n+\omega_n^2q_n=-\Gamma_n\ddot{u}_g(t)$$

Modal Response

$$\mathbf{u}_n(t)=\phi_nq_n(t)=\Gamma_n\phi_nD_n(t)$$

$$\mathbf{f}_n(t)=\mathbf{s}_nA_n(t) \qquad A_n(t)=\omega_n^2D_n(t)$$

$$r_n(t)=r_n^{st}A_n(t)$$

Modal Static Response

Forces: by statics

Displacements: $\mathbf{u}_n^{st}=\dfrac{\Gamma_n}{\omega_n^2}\phi_n$

Total Response

$$r(t)=\sum_{n=1}^{N}r_n(t)=\sum_{n=1}^{N}r_n^{st}A_n(t)$$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\mathbf{u}_n(t)=\sum_{n=1}^{N}\Gamma_n\phi_nD_n(t)$$

For Multistory Building with Symmetric Plan

$$\Gamma_n=\frac{L_n^h}{M_n} \qquad L_n^h=\sum_{j=1}^{N}m_j\phi_{jn} \qquad M_n=\sum m_j\phi_{jn}^2$$

$$\mathbf{s}_n=\Gamma_n\mathbf{m}\,\phi_n \qquad s_{jn}=\Gamma_nm_j\phi_{jn}$$

Effective Modal Mass and Height

$$M_n^*=\Gamma_nL_n^h=\frac{\left(L_n^h\right)^2}{M_n} \qquad h_n^*=\frac{L_n^\theta}{L_n^h} \qquad L_n^\theta=\sum_{j=1}^{N}h_jm_j\phi_{jn}$$

$$\sum_{n=1}^{N}M_n^*=\sum_{j=1}^{N}m_j \qquad \sum_{n=1}^{N}h_n^*M_n^*=\sum_{j=1}^{N}h_jm_j$$

Total Response

$$r(t)=\sum_{n=1}^{N}r_n(t)=\sum_{n=1}^{N}r_n^{st}A_n(t)$$

Earthquake Response Spectrum Analysis

$$r_{no}=r_n^{st}A_n \qquad r_o\equiv\max_t|r(t)| \qquad r_o\cong\left(\sum_{n=1}^{N}r_{no}^2\right)^{1/2}$$

UNIVERSITY OF CALIFORNIA, BERKELEY
Spring Semester 2021

Dept. of Civil and Environmental Engineering
Structural Engineering, Mechanics and Materials

Name: ____________________________________

Ph.D. Preliminary Examination
Dynamics

Two pages of useful formulas are attached at the end of the exam.

Question 1 (40% weight)

The structure shown in Fig. 1(a) is subjected to a free-vibration experiment, results of which are shown in Fig. 1(b).

i) Estimate the natural period and damping ratio.

ii) For the spectrum shown in Fig. 1(c), but scaled to 0.8g PGA, determine the base shear coefficient of the structure.

iii) The displacement of the beam should not exceed 0.025 m. If a linear viscous damper is installed diagonally in the frame to ensure that the drift limit is not exceeded, determine its minimum damping coefficient c_d.

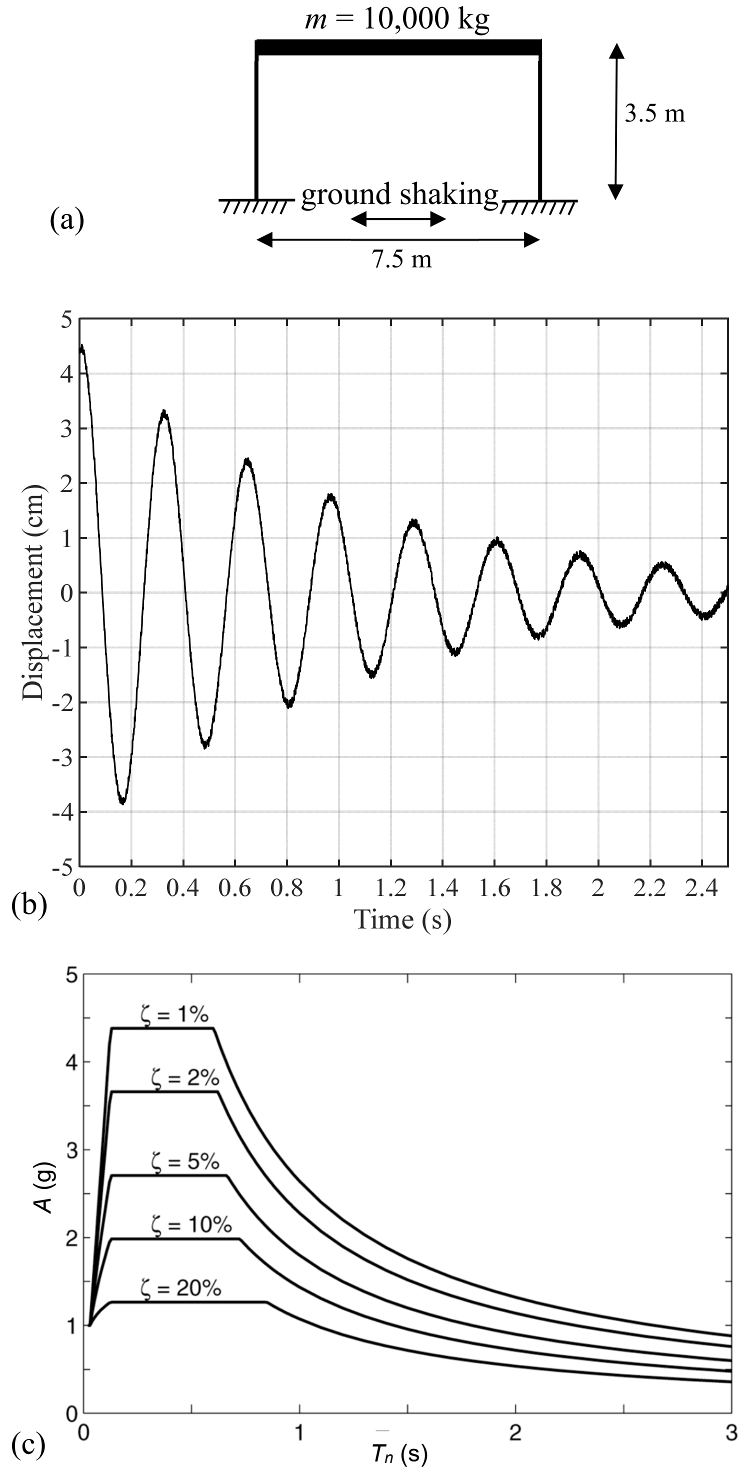


Figure 1

Question 2 (60% weight)

The dynamics of a vehicle crossing a single-span bridge can be idealized using the model shown in Fig. 2. The vehicle is represented by the mass-spring-dashpot system with properties m_v, k, and c. The suspension system remains vertical and in contact with the road at all times. The bridge deck of length L is represented by a beam with uniform flexural rigidity EI and uniform mass per unit length $\bar{m}$. The vehicle is traveling with a constant speed v. If the bridge deck displacement is given by $u_b(x,t) = \psi(x)z(t)$, where the vibration shape is assumed to be $\psi(x) = \sin(\pi x / L)$, and that the absolute vertical displacement of the vehicle mass is $u_v(t)$, obtain the equations of motion for the system while the vehicle is on the bridge.

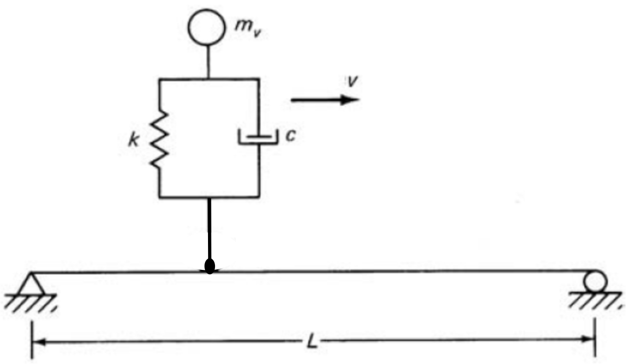


Figure 2

Useful expressions:

For the deck: $u_b(x,t) = \psi(x)z(t)$

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx \qquad \tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx$$

$$\tilde{p}(t) = \int_0^L p(x,t)\psi(x)dx \qquad \int_0^L \sin^2(\pi x / L)dx = L/2$$

Basic Definitions and Equations

$$\omega_n = \sqrt{\frac{k}{m}} \qquad T_n = \frac{2\pi}{\omega_n} \qquad f_n = \frac{1}{T_n} \qquad (u_{st})_o = \frac{p_o}{k}$$

$$\varsigma = \frac{c}{2m\omega_n} = \frac{c}{c_{cr}}$$

$\varsigma > 1$ Overdamped

$\varsigma = 1$ Critically damped

$\varsigma < 1$ Underdamped

$$\omega_D = \omega_n\sqrt{1-\varsigma^2}$$

Free Vibration $m\ddot{u} + c\dot{u} + ku = 0$

For $\varsigma = 0$: $\quad u(t) = u(0)\cos\omega_n t + \dfrac{\dot{u}(0)}{\omega_n}\sin\omega_n t$

For $0 < \varsigma < 1$:

$$u(t) = e^{-\varsigma\omega_n t}\left(u(0)\cos\omega_D t + \frac{\dot{u}(0) + \varsigma\omega_n u(0)}{\omega_D}\sin\omega_D t\right)$$

Decay of Motion

$$\frac{u_1}{u_{j+1}} = \exp\left(\frac{2j\pi\varsigma}{\sqrt{1-\varsigma^2}}\right)$$

Free Vibration Test

$$\varsigma = \frac{1}{2\pi j}\ln\frac{u_i}{u_{i+j}}$$

Harmonic Excitation $m\ddot{u} + c\dot{u} + ku = p_o\sin\omega t$

Steady State Response

$$u(t) = u_o\sin(\omega t - \phi)$$

$$R_d = \frac{u_o}{(u_{st})_o} = \frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + [2\zeta(\omega/\omega_n)]^2}}$$

$$\varphi = \tan^{-1}\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}$$

Resonance at $\omega_n\sqrt{1-2\zeta^2}$ with $R_d = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

Half-Power Bandwidth

$$\frac{\omega_b - \omega_a}{\omega_n} = 2\zeta$$

Vibration Generator: $\quad p(t) = (m_e e\omega^2)\sin\omega t$

Transmissibility $\quad TR = (f_T)_o / p_o = \ddot{u}_o^t / \ddot{u}_{go}$

$$TR = R_d\sqrt{1 + [2\zeta(\omega/\omega_n)]^2}$$

Equivalent Viscous Damping

$$\zeta_{eq} = \frac{1}{4\pi}\frac{E_D}{E_{So}}$$

Arbitrary Excitation $m\ddot{u} + c\dot{u} + ku = p(t)$

Reponse to unit Impulse: $\quad p(t) = \delta(t-\tau)$

$$h(t-\tau) \equiv u(t) = \frac{1}{m\omega_n}\sin(\omega_n(t-\tau)) \qquad \zeta = 0$$

Duhamel's Integral

$$u(t) = \int_0^t p(\tau)h(t-\tau)d\tau$$

Response to Step Force, $\varsigma = 0$

$$u(t) = (u_{st})_o(1-\cos\omega_n t)$$

Response to Ramp Force, $\varsigma = 0$

$$p(t) = p_o\frac{t}{t_r} \qquad u(t) = (u_{st})_o\left(\frac{t}{t_r} - \frac{\sin\omega_n t}{\omega_n t_r}\right)$$

Response to Rectangular Pulse

$$R_d = \begin{cases} 2\sin\pi t_d/T_n & t_d/T_n \le \frac{1}{2} \\ 2 & t_d/T_n \ge \frac{1}{2} \end{cases}$$

Short Pulse $\quad I = \int p(t)dt \qquad u(t) = I\left(\dfrac{1}{m\omega_n}\sin\omega_n t\right)$

Earthquake Response $p(t) = p_{eff}(t) = -m\ddot{u}_g(t)$

$$u \equiv u(t, T_n, \zeta) \qquad u_o \equiv u_o(T_n, \zeta)$$

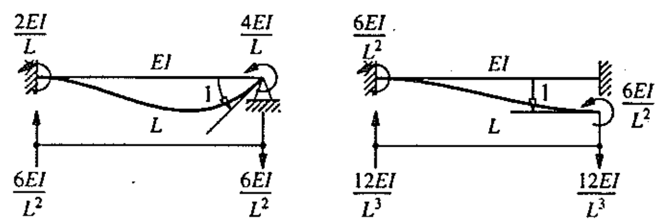
$$D \equiv u_o \qquad V = \omega_n D \qquad A = \omega_n^2 D$$

$$\frac{A}{\omega_n} = V = \omega_n D \qquad E_{so} = \frac{mV^2}{2} \qquad f_{so} = kD = mA$$

For One Story Structure

$$V_{bo} = f_{so} = \frac{A}{g}w \qquad M_{bo} = hV_{bo}$$

Stiffness Coefficients for a Flexural Element



EQ Reponse of Inelastic Systems $m\ddot{u} + c\dot{u} + f_s(u,\dot{u}) = p_{eff}(t) = -m\ddot{u}_g(t)$

Normalized Yield Strength

$$\bar{f}_y = \frac{f_y}{f_o} = \frac{u_y}{u_o}$$

f_y and u_y are yield strength and yield deformation

f_o and u_o are peak force and deformation in corresponding linear system

Yield Strength Reduction Factor

$$R_y = \frac{f_o}{f_y} = \frac{u_o}{u_y} \qquad f_o = ku_o$$

f_o is minimum strength required for structure to remain elastic

Ductility Factor

$$\mu = \frac{u_m}{u_y}$$

u_m is peak deformation of elastoplastic system

Response Spectrum for Inelastic Systems

$$D_y = u_y \qquad V_y = \omega_n D_y \qquad A_y = \omega_n^2 D_y$$

$$f_y = \frac{A_y}{g}w \qquad u_m = \mu\left(\frac{T_n}{2\pi}\right)^2 A_y$$

Generalized SDOF Systems: Distributed Mass and Elasticity

For Assumed Shape Function $\psi(x)$

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 dx \qquad \tilde{\Gamma} = \frac{\tilde{L}}{\tilde{m}} \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 dx \qquad \omega_n^2 = \frac{\tilde{k}}{\tilde{m}}$$

$$\tilde{L} = \int_0^L m(x)\psi(x)dx \qquad z_o = \tilde{\Gamma}D$$

At Height x Above the Base

$$u_o(x) = \tilde{\Gamma}D\psi(x) \qquad f_o(x) = \tilde{\Gamma}m(x)\psi(x)A$$

Static Analysis of the tower due to $f_o(x)$ provides interal forces.

Base Shear and Moment:

$$V_{bo} = V_o(0) = \tilde{L}\tilde{\Gamma}A \qquad M_{bo} = M_o(0) = \tilde{L}^\theta\tilde{\Gamma}A$$

$$\tilde{L}^\theta = \int_0^L xm(x)\psi(x)dx$$

Equation of Motion, MDOF $\quad \mathbf{m\ddot{u}+c\dot{u}+ku=p}(t)$

Earthquake Excitation

$$\mathbf{p}(t)=\mathbf{p}_{eff}(t)=-\mathbf{m}\,\iota\,\ddot{u}_g(t)$$

where $\iota \equiv$ influence vector

Static Condesation

$\mathbf{u}_t \equiv$ translational DOF $\;\mathbf{u}_o \equiv$ rotational DOF

$$\begin{bmatrix}\mathbf{m}_{tt} & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{u}}_t\\ \ddot{\mathbf{u}}_o\end{Bmatrix}+\begin{bmatrix}\mathbf{k}_{tt} & \mathbf{k}_{to}\\ \mathbf{k}_{ot} & \mathbf{k}_{oo}\end{bmatrix}\begin{Bmatrix}\mathbf{u}_t\\ \mathbf{u}_o\end{Bmatrix}=\begin{Bmatrix}\mathbf{p}_t(t)\\ \mathbf{0}\end{Bmatrix}$$

$$\hat{\mathbf{k}}_{tt}=\mathbf{k}_{tt}-\mathbf{k}_{ot}^T\mathbf{k}_{oo}^{-1}\mathbf{k}_{ot}$$

To find $\mathbf{u}_t(t)$ solve: $\mathbf{m}_{tt}\ddot{\mathbf{u}}_t+\hat{\mathbf{k}}_{tt}\mathbf{u}_t=\mathbf{p}_t(t)$

To find $\mathbf{u}_o(t)$ use: $\mathbf{u}_o=-\mathbf{k}_{oo}^{-1}\mathbf{k}_{ot}\mathbf{u}_t$

Natural Frequencies and Modes

To find ω_n^2, use characteristic equation: $\det\left[\mathbf{k}-\omega_n^2\mathbf{m}\right]=0$

To find ϕ_n, solve: $\left[\mathbf{k}-\omega_n^2\mathbf{m}\right]\phi_n=0$

Orthogonality of Modes

$$\phi_n^T\mathbf{k}\,\phi_r=0 \qquad \phi_n^T\mathbf{m}\,\phi_r=0$$

Modal Expansion of Displacements

$$\mathbf{u}=\sum_{r=1}^{N}\phi_r q_r=\Phi\mathbf{q} \qquad \text{where: } q_n=\frac{\phi_n^T\mathbf{m}\mathbf{u}}{M_n}$$

For Initial Conditions: $\mathbf{u}(0),\dot{\mathbf{u}}(0)$

$$q_n(0)=\frac{\phi_n^T\mathbf{m}\,\mathbf{u}(0)}{M_n} \qquad \dot{q}_n(0)=\frac{\phi_n^T\mathbf{m}\,\dot{\mathbf{u}}(0)}{M_n}$$

Modal Equations

$$M_n\ddot{q}_n+C_n\dot{q}_n+K_nq_n=P_n(t)$$

$$\ddot{q}_n+2\zeta_n\omega_n\dot{q}_n+\omega_n^2q_n=\frac{P_n(t)}{M_n}$$

$$K_n=\phi_n^T\mathbf{k}\,\phi_n \qquad M_n=\phi_n^T\mathbf{m}\,\phi_n \qquad P_n(t)=\phi_n^T\mathbf{p}(t)$$

For classical damping: $C_n=\phi_n^T\mathbf{c}\,\phi_n \qquad \varsigma_n=\dfrac{C_n}{2M_n\omega_n}$

Dynamic Response

Solve modal equations for $q_n(t)$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\mathbf{u}_n(t)=\sum_{n=1}^{N}\boldsymbol{\varphi}_nq_n(t)$$

Equivalent static forces, nth mode

$$\mathbf{f}_n(t)=\omega_n^2\mathbf{m}\boldsymbol{\varphi}_nq_n(t)$$

Internal forces by static analysis

EQ Response History Analysis $\quad \mathbf{m\ddot{u}+c\dot{u}+ku=p}_{eff}(t)=-\mathbf{m}\,\iota\,\ddot{u}_g(t)$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\phi_nq_n(t) \qquad \mathbf{m}\,\iota=\sum_{n=1}^{N}s_n=\sum_{n=1}^{N}\Gamma_n\mathbf{m}\,\phi_n$$

$$\Gamma_n=\frac{L_n}{M_n} \qquad L_n=\phi_n^T\mathbf{m}\,\iota \qquad M_n=\phi_n^T\mathbf{m}\,\phi_n$$

$$q_n(t)=\Gamma_nD_n(t)$$

Modal Equations for Classically-Damped Systems

$$\ddot{q}_n+2\varsigma_n\omega_n\dot{q}_n+\omega_n^2q_n=-\Gamma_n\ddot{u}_g(t)$$

Modal Response

$$\mathbf{u}_n(t)=\phi_nq_n(t)=\Gamma_n\phi_nD_n(t)$$

$$\mathbf{f}_n(t)=\mathbf{s}_nA_n(t) \qquad A_n(t)=\omega_n^2D_n(t)$$

$$r_n(t)=r_n^{st}A_n(t)$$

Modal Static Response

Forces: by statics

Displacements: $\mathbf{u}_n^{st}=\dfrac{\Gamma_n}{\omega_n^2}\phi_n$

Total Response

$$r(t)=\sum_{n=1}^{N}r_n(t)=\sum_{n=1}^{N}r_n^{st}A_n(t)$$

$$\mathbf{u}(t)=\sum_{n=1}^{N}\mathbf{u}_n(t)=\sum_{n=1}^{N}\Gamma_n\phi_nD_n(t)$$

For Multistory Building with Symmetric Plan

$$\Gamma_n=\frac{L_n^h}{M_n} \qquad L_n^h=\sum_{j=1}^{N}m_j\phi_{jn} \qquad M_n=\sum m_j\phi_{jn}^2$$

$$\mathbf{s}_n=\Gamma_n\mathbf{m}\,\phi_n \qquad s_{jn}=\Gamma_nm_j\phi_{jn}$$

Effective Modal Mass and Height

$$M_n^*=\Gamma_nL_n^h=\frac{\left(L_n^h\right)^2}{M_n} \qquad h_n^*=\frac{L_n^\theta}{L_n^h} \qquad L_n^\theta=\sum_{j=1}^{N}h_jm_j\phi_{jn}$$

$$\sum_{n=1}^{N}M_n^*=\sum_{j=1}^{N}m_j \qquad \sum_{n=1}^{N}h_n^*M_n^*=\sum_{j=1}^{N}h_jm_j$$

Total Response

$$r(t)=\sum_{n=1}^{N}r_n(t)=\sum_{n=1}^{N}r_n^{st}A_n(t)$$

Earthquake Response Spectrum Analysis

$$r_{no}=r_n^{st}A_n \qquad r_o\equiv\max_t|r(t)| \qquad r_o\cong\left(\sum_{n=1}^{N}r_{no}^2\right)^{1/2}$$

Name: ______________________________

University of California, Berkeley
Civil and Environmental Engineering

Structural Engineering, Mechanics & Materials
Spring Semester, 2020

Comprehensive Examination - Dynamics

Problem 1 (50% weight)
A single-degree-of-freedom oscillator has a mass of 100 lb, a natural period of 0.5 seconds, and a fraction of critical damping of 5%. The oscillator is subjected to a harmonic ground motion with an amplitude of 0.1g and a frequency of 4 Hz.

Determine the maximum total displacement of the mass.

Problem 2 (50% weight)
Figure 1 shows a one-story portal frame. Assume the floor remains horizontal during motion. The floor weight is W = 80 kips. The columns have a height of h = 12 ft and a flexural rigidity of EI = 2 x 10^6 lb-ft^2. Assume that ζ = 5%, the force–deformation relation is elastoplastic, the design earthquake has a peak ground acceleration of 0.6g, and the elastic design spectrum in Fig. 6.9.5 applies (after scaling to the correct PGA).

Determine the lateral force for which the frame should be designed if:

(a) the system is required to remain elastic
(b) the allowable ductility factor is 4.

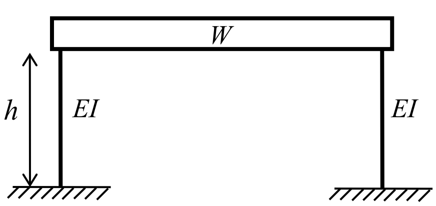


Figure 1

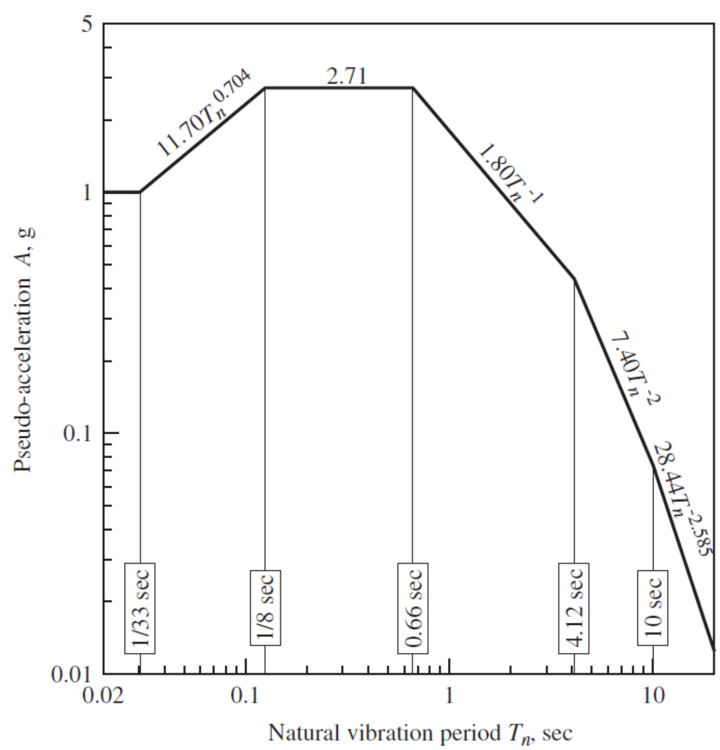


Figure 6.9.5 Elastic pseudo-acceleration design spectrum (84.1th percentile) for ground motions with $\ddot{u}_{go} = 1$g, $\dot{u}_{go} = 48$ in./sec, and $u_{go} = 36$ in.; $\zeta = 5\%$.

Name: ______________________________

University of California, Berkeley
Civil and Environmental Engineering

Structural Engineering, Mechanics & Materials
Spring Semester, 2019

Comprehensive Examination - Dynamics

Problem 1 (30% weight)
Consider an industrial machine that weighs 1000 lbs and is supported on spring-type isolators of total stiffness $k = 3000$ lb/ft. The machine operates at a frequency of $f = 2$ Hz with a force unbalance of $p_o = 100$ lbs. Assume 5% damping.

(a) Determine the maximum displacement of the machine (from the at rest position) during steady state oscillation.

Problem 2 (30% weight)
The elevated water tank of Figure 1a weighs 90 kips. The tower has a lateral stiffness of 10 kips/in, and is subjected to the time-varying force $p(t)$ shown in Figure 1b. Assume zero damping and that the tower is at rest at time $t = 0$. Treating the water tower as an SDOF system, determine:

(a) the equation that describes the dynamic response $u(t)$ for $0 < t < 4$ seconds.
(b) the equation that describes the dynamic response $u(t)$ for $t > 4$ seconds.

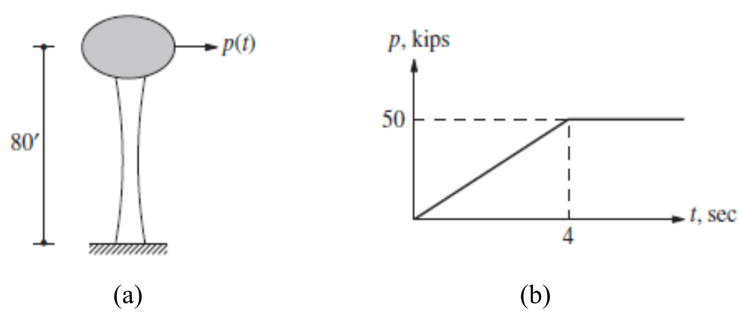


Figure 1

Problem 3 (40% weight)

Figure 2 shows a two-storey sway-frame. Assume the floors remain horizontal during motion. The weight of each storey is W = 100 kips. All columns have a height of h = 12 ft and a flexural rigidity of $EI = 5 \times 10^6$ lb-ft^2.

(a) Assuming zero damping, write the equation of motion.

(b) Assuming zero damping, the natural vibration periods are 2.15 seconds and 0.82 seconds. Determine the mode shapes.

(c) Assuming 2% damping, use the response spectra in Figure 3 to estimate the maximum acceleration of the top storey.

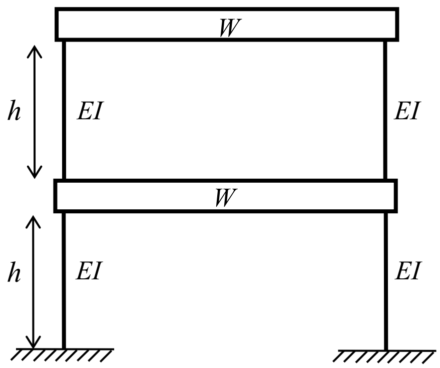
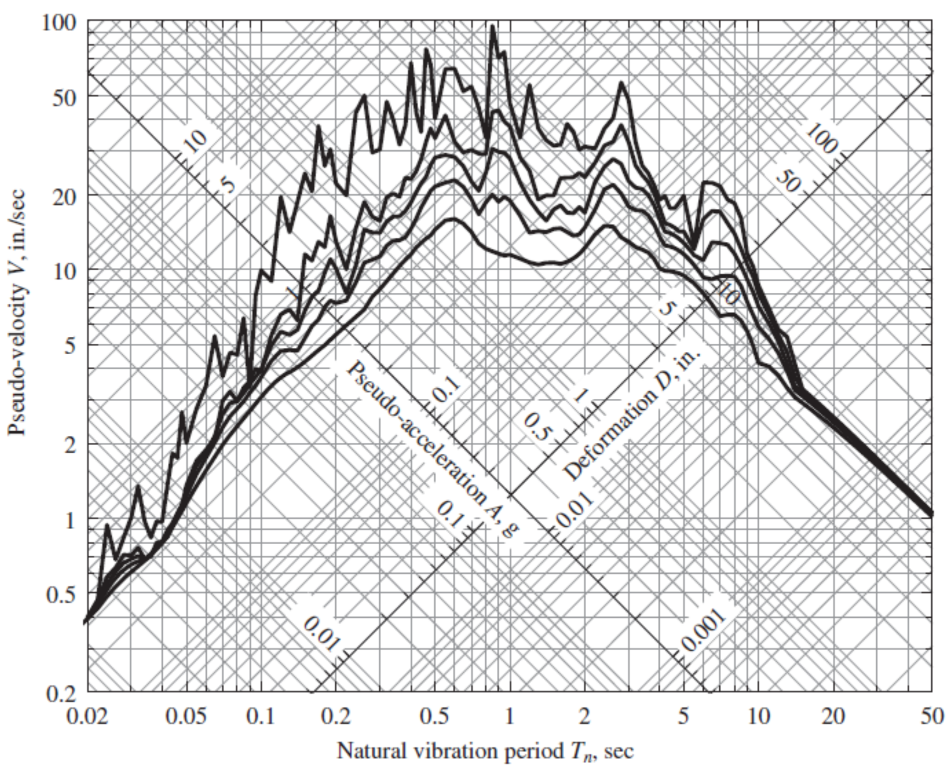


Figure 2



* Spectra shown for ζ = 0, 2, 5, 10, 20%

Figure 3