Mechanics
PhD Prelim Exam Spring 2020

All questions have equal weighting.

1. Consider single crystal composed of cubic Manganese, `Mn`. In a basis $\{\boldsymbol{e}_i\}$ aligned with the crystal's cubic axes the compliance components in Voigt notation are

$$\mathbb{S} = \begin{bmatrix} 5.6 & -2.2 & -2.2 & 0 & 0 & 0 \\ -2.2 & 5.6 & -2.2 & 0 & 0 & 0 \\ -2.2 & -2.2 & 5.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10.1 \end{bmatrix} [1/\mathrm{TPa}] \, .$$

   What is the Young's modulus for the material in the direction $\boldsymbol{d} = (\boldsymbol{e}_1 + \boldsymbol{e}_2)/\sqrt{2}$?

2. At a given point $\boldsymbol{x}_o$ in a body $\mathcal{B}$, the traction is known to be

$$\boldsymbol{t} = 1\boldsymbol{e}_1 + 2\boldsymbol{e}_2 \text{ MPa}$$

   on the plane with normal $\boldsymbol{n} = \boldsymbol{e}_1$; it is known to be

$$\boldsymbol{t} = 2\boldsymbol{e}_2 + 3\boldsymbol{e}_3 \text{ MPa}$$

   on the plane with normal $\boldsymbol{n} = \boldsymbol{e}_3$; and it is known to be

$$\boldsymbol{t} = 2/\sqrt{2}\boldsymbol{e}_1 + 2\boldsymbol{e}_2 + 5/\sqrt{2}\boldsymbol{e}_3 \text{ MPa}$$

   on the plane with normal $\boldsymbol{n} = (\boldsymbol{e}_2 + \boldsymbol{e}_3)/\sqrt{2}$. What is the state of stress at $\boldsymbol{x}_o$

3. <u>Starting</u> from the strong form of the equilibrium equations and assuming small strain, show that power of the external loads on a body $\mathcal{B}$

$$\mathcal{P}_{\text{ext}} = \int_{\partial\mathcal{B}} t_i v_i \, dA + \int_{\mathcal{B}} b_i v_i \, dV$$

   is equal to the stress power plus the rate of change of the kinetic energy

$$\mathcal{P}_{\text{int}} = \int_{\mathcal{B}} \left(\dot{\epsilon}_{ij}\sigma_{ij} + \dot{\overline{\tfrac{1}{2}\rho v^2}} \right) \, dV \, ,$$

   where v_i are the velocity components, ϵ_{ij} are the small strain components, σ_{ij} are the stress components, ρ is the mass density, t_i are the traction components, and b_i are the volumetric body force components (force/volume). You must justify each step of your derivation with a short statement.

Mechanics
PhD Preliminary Fall 2019

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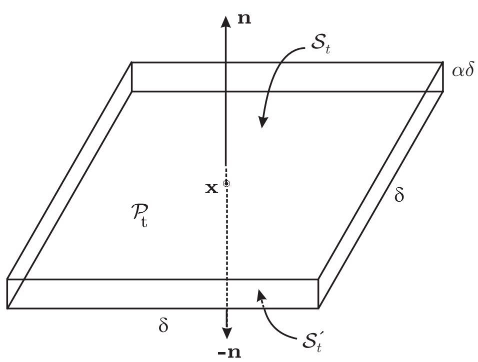
1. Linear elastic monoclinic materials have a single plane of reflective symmetry. Show that such materials have only 13 (independent) non-zero elastic moduli.

2. Assuming only global linear and angular momentum balance, show using a *pill-box* construction, that the traction vector on a surface with normal $\boldsymbol{n}$ at any point in a body $\mathcal{B}$ is an odd function of the surface normal, i.e. show

$$\boldsymbol{t}(\boldsymbol{x}, \boldsymbol{n}) = -\boldsymbol{t}(\boldsymbol{x}, -\boldsymbol{n}) \qquad \forall \boldsymbol{x} \in \mathcal{B}\,, \quad \forall \boldsymbol{n}\,. \tag{1}$$



3. Consider a bolt of diameter D and assume that the bolt is made from a linear viscoelastic material with *given* stress relaxation modulus $E_r(t)$. The bolt is used to join two *rigid* plates. An impact drill is used to tighten the bolt very quickly and the instantaneous force in the bolt at time $t = 0^+$ is known to be P_o.

   (a) Find an expression for the strain in the bolt in terms of P_o and $E_r(\cdot)$.

   (b) Find an expression for the force in the bolt at any time $t > 0$ in terms of $E_r(\cdot)$ and your answer to Part 3a.

   Treat this problem as one-dimensional. Ignore all three-dimensional aspects of the problem.

Mechanics
PhD Preliminary Fall 2018

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1. (10 points) Consider the problem of small strain, isotropic, linear elasticity for two-dimensional plane strain. Show that the compatibility equation given in terms of the Airy stress function can be written as

$$\nabla^4 \phi = 0\,, \tag{1}$$

where in Cartesian form $\nabla^4 \phi = \phi_{,1111} + 2\phi_{,1212} + \phi_{,2222}$. Recall also by definition that

$$\sigma_{11} = \phi_{,22} \tag{2}$$

$$\sigma_{22} = \phi_{,11} \tag{3}$$

$$\sigma_{12} = -\phi_{,12}\,, \tag{4}$$

and $\nabla \times \nabla \times \boldsymbol{\epsilon} = 0$, which in Cartesian form is given as $S_{pq} = e_{pki} e_{qlj} \epsilon_{ij,kl} = 0$.

2. (10 points) For the motion

$$\boldsymbol{x} = X_1 \boldsymbol{e}_1 + (bX_1 + X_2)\boldsymbol{e}_2 + (X_3 + \frac{1}{2}c(X_3)^2)\boldsymbol{e}_3 \tag{5}$$

with given constants b and c, determine (i) the deformation gradient field $\boldsymbol{F}(\boldsymbol{X})$, (ii) the extremal stretches at the point $(X_1, X_2, X_3) = (1, 1, 1)$, and (iii) the volumetric strain as this point.

3. (10 points) Consider a material point whose stress state is given by

$$\boldsymbol{\sigma}(t) \sim \begin{bmatrix} 0 & a \cdot t & 0 \\ a \cdot t & 0 & 0 \\ 0 & 0 & a \cdot t \end{bmatrix}. \tag{6}$$

Assume the material to be a ductile metal governed by the Mises yield condition $f(\boldsymbol{\sigma}) = \|\boldsymbol{\sigma}'\| - \sqrt{\frac{2}{3}}\sigma_Y$, where $\boldsymbol{\sigma}'$ denotes the stress deviator and σ_Y is the material's yield stress. (i) Find an expression for the time of initial yield; (ii) Assuming associative plastic flow (Prandtl-Reuss), determine the direction of plastic flow at the moment of yield; (iii) Can the given loading be sustained by the material if material does not harden?

Mechanics
PhD Preliminary Spring 2017

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1. (10 points) Consider a body Ω that is assembled by gluing together two separate bodies along a flat interface. The normal vector to the interface is given by $\boldsymbol{n} = (1\ 1\ 1)^T$. Assume that Ω is subjected to a homogeneous (Cauchy) stress field

$$\boldsymbol{\sigma} = \beta \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} \text{ MPa}$$

where β is a non-dimensional load factor. Assume that the interface fails when the normal stress on the interface exceeds 100 MPa. Determine the maximal permissible value of β.

2. (10 points) Consider a small-strain hyperelastic body Ω, free of body forces, with strain energy density $W(\boldsymbol{\varepsilon})$ subjected to dead-loads $\bar{\boldsymbol{t}}$ on $\partial\Omega_t \subset \partial\Omega$ and restrained with zero displacement on $\partial\Omega_u \subset \partial\Omega$, where $\overline{\partial\Omega_t \cap \partial\Omega_u} = \partial\Omega$. Starting from the principal of minimum potential energy, derive the strong form of the governing equilibrium equations.

3. (10 points) A three dimensional viscoelastic body $\Omega = \{\boldsymbol{x} \mid x_1^2 + x_2^2 \leq R^2 \text{ and } |x_3| \leq L/2\}$ has a measured displacement field $\boldsymbol{u} = (\alpha x_1/R)\boldsymbol{e}_1 + (\alpha x_2/R)\boldsymbol{e}_2$, where R and L are given and $\alpha > 0$ is much smaller than R and L. Determine the algebraically maximum and minimum normal strains in the body as well as the maximum shear strain in the body.
