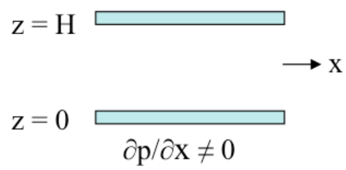
Comprehensive exam - CE 200A - Environmental fluid mechanics I

Consider channel flow between two parallel plates driven by a constant pressure gradient. You will solve for the steady-state velocity profile in this uni-directional flow, which is assumed to be two dimensional. Then you will consider how a scalar evolves in this flow field.



1. Label the terms in the Navier-Stokes equations (given below).
2. State the appropriate boundary conditions for the Navier-Stokes equations for this problem.
3. Simplify the Navier-Stokes equations based on the geometry and conditions of this problem. You do not need to a do formal scaling, but justify all your steps.
4. Solve for the velocity field and sketch the velocity profile.
5. Now state the time-dependent scalar advection-diffusion equation in 1D, considering just the streamwise direction, x, and accounting for the flow solution you obtained above. [You can use the velocity field you obtained above as a given (known solution), then find the average velocity needed for advection in x.]
6. Discretize the scalar transport equation using central differences in space and explicit Euler in time. (Do you remember if this scheme is stable?)
7. State the name of another discretization method that could be used to make the solution second order accurate in both time and space and explain why you would choose it.
8. Sketch a picture of the scalar field at different times.

Incompressible Navier-Stokes equations in index notation with continuity in Cartesian coordinates:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - g\delta_{i3} \tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

Scalar transport equation in index notation:

$$\frac{\partial c}{\partial t} + u_i \frac{\partial c}{\partial x_i} = \kappa \frac{\partial^2 c}{\partial x_i \partial x_i} \tag{3}$$